Quiz 4 Key

1. Let $P(1,3,2)$ $Q(3,-1,6)$ $R(5,2,0)$ and $S(3,6,1.5)$ $T(-9,-14,-12.5)$

a)[3] Find an equation for a plane that passes through P, Q, and R

(+1.5) $\vec{PQ} = \langle 3-1, -1-3, 6-2\rangle = \langle 2,-4,4\rangle$

$\vec{PR} = \langle 5-1, 2-3, 0-2\rangle = \langle 4,-1,-2\rangle$

(+1) $\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 2 & -4 & 4 \\ 4 & -1 & -2 \end{vmatrix} = \vec{i}(8+4) - \vec{j}(-4-16) + \vec{k}(-2+16)$

$= 12\vec{i} + 20\vec{j} + 14\vec{k}$

(+1) so $\langle 12, 20, 14\rangle$ is normal to the plane

$10 = 12(x-1) + 20(y-3) + 14(z-2)$

work

skeleton (+.5)

b) [3] Does a line that passes through S and T intersect the plane you found in part (a)? If so, find the point it does so.

line through S & T:

direction $\vec{ST} = \langle -9-3, -14-6, -12.5-1.5\rangle$

$= \langle -12, -20, -14\rangle$

(+.5) $\vec{ST} \parallel \vec{n}$ so $\vec{ST}$ will intersect the plane & form a $\perp$

reason (+.5)

(+.5) Yes

(+1) the line is

(+.5) $\langle 3,6,1.5\rangle + t\langle -12,-20,-14\rangle$

$\Rightarrow x = 3-12t \quad y = 6-20t \quad z = 1.5-14t$

(+.5) we are looking for the time that the line intersects the plane ie satisfies the eqn of the plane

$0 = 12(3-12t) + 20(6-20t-3) + 14(1.5-14t-2)$

$\Rightarrow t = 38.5/185$ so at $\left(3-12\cdot\frac{38.5}{185}, 6-20\cdot\frac{38.5}{185}, 1.5-14\cdot\frac{38.5}{185}\right)$

2.[6] Let $f(x,y) = e^{-(x^2+y^2)} = e^{-x^2-y^2}$. Find the following

a) $\frac{df}{dx} = e^{-x^2-y^2}\cdot -2x$

(+1) $= -2xe^{-x^2-y^2}$

b) $f_y(x,y) = e^{-x^2-y^2}\cdot -2y$

(+1) $= -2ye^{-x^2-y^2}$

c) $f_{xy}(x,y) = -2xe^{-x^2-y^2}\cdot -2y$

(+2) $= 4xye^{-x^2-y^2}$

d) $\frac{d^2f}{dy^2} = -2ye^{-x^2-y^2}\cdot -2y + -2e^{-x^2-y^2}$

product (+1)

$= 4y^2e^{-x^2-y^2} - 2e^{-x^2-y^2}$

correct y (+.5)

start with (+.5)

3. Match the following equations to their respective graphs.

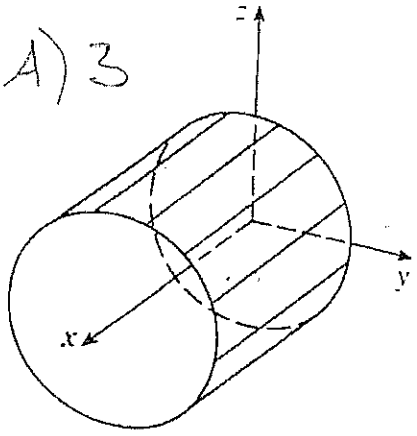
a)[4] 1) $z = y^2 - x^2$ C

2) $z = \sin x + \sin y$ D

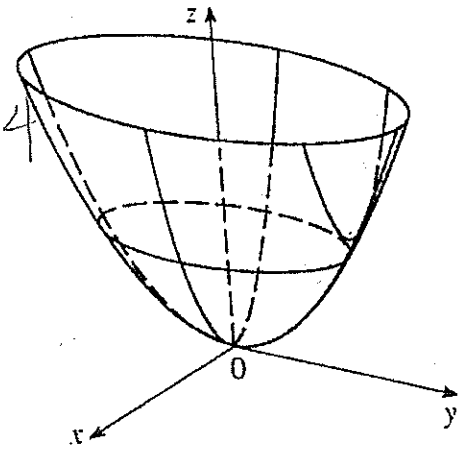
3) $z^2 = 1 - y^2$ A
x is not involved

4) $z = 4\sqrt{x^2 + y^2}$ B

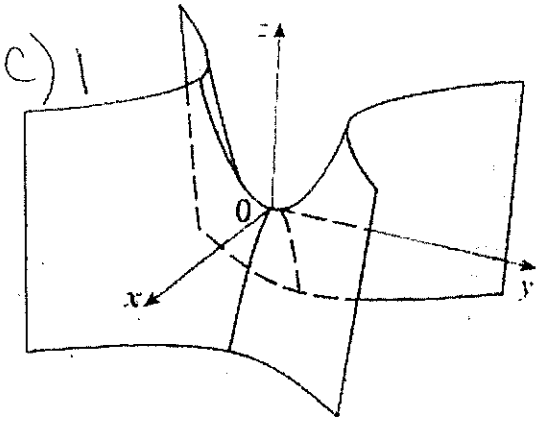
A) 3



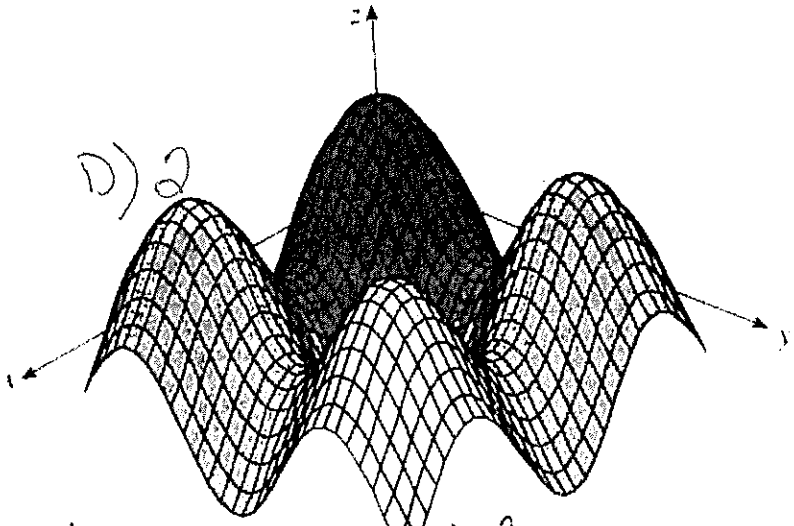
B) 4



C) 1



D) 2



b)[2] Which of the above graphs are graphs of functions where z is a function of x and y.

B, C & D 1, 2 & 4

4. [2] Find the domain of $f(x,y) = \sqrt{1-x^2} - \sqrt{1-y^2}$.

need (+.5) $1 - x^2 \geq 0$ and (+.5) $1 - y^2 \geq 0$

$1 \geq x^2$ $1 \geq y^2$

(+.5) $-1 \leq x \leq 1$ $-1 \leq y \leq 1$

s.o. $\{(x,y) \in \mathbb{R}^2 \mid -1 \leq x \leq 1 \text{ & } -1 \leq y \leq 1\}$

ie a block like (+.5)