TMATH 126: Quiz 3

Key

You may use any work of yours that you made from last week. This includes, practice problems from the book and worked out WebAssign problems. This *does not* include photocopies of notes from the book or tutorials shown on WebAssign. You may also use a calculator, but you are not allowed to use any device that can access the internet.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide either counterexample or reasoning for your answer.

   Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be vectors in $\mathbb{R}$.
   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

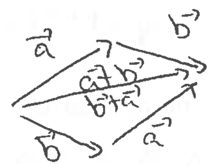
(T) F $\vec{a} + \vec{b} = \vec{b} + \vec{a}$.

+.5

started +.5

reason +1

by the 'triangle prop' just dif sides of the same parallelogram.



T (F) $(\vec{a} \cdot \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \cdot \vec{c})$.

this makes no sense $\vec{a} \cdot \vec{b}$ is a # but we can't dot a # with the vector $\vec{c}$.

T (F) $(\vec{a} \cdot \vec{b}) \times \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c})$.

this makes no sense $\vec{a} \cdot \vec{b}$ is a # but we can't cross a # with a vector $\vec{c}$.

(T) F The vectors $\langle 4, -8 \rangle$ and $\langle 2, 1 \rangle$ are perpendicular.

$$\frac{\langle 4,-8\rangle \cdot \langle 2,1\rangle}{\|\langle 4,-8\rangle\| \, \|\langle 2,1\rangle\|} = \frac{8-8=0}{\sqrt{16+64}\sqrt{5}} = \cos\theta$$

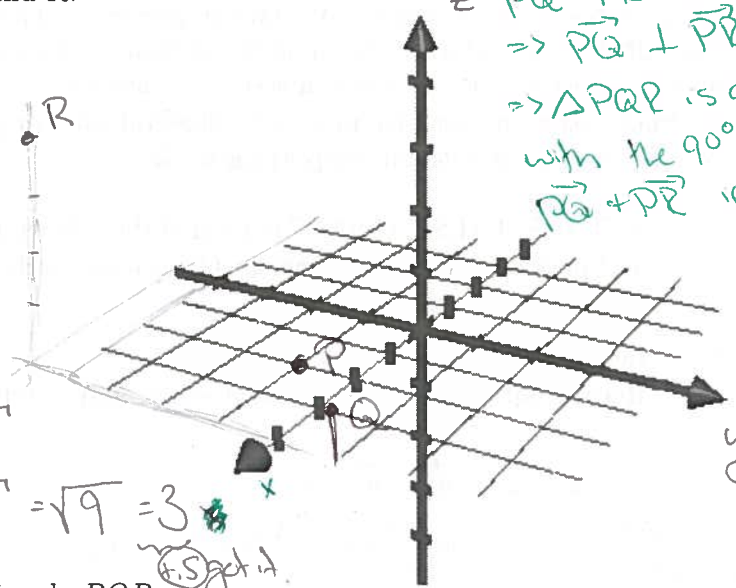
$\Rightarrow \theta$ is $\pi/2$ ie $90°$

$$\begin{array}{r} 36 \\ 4\overline{)144} \\ -12 \\ 24 \end{array} \qquad \begin{array}{r} 72 \\ 9 \\ \hline 81 \end{array}$$

2. Consider the points: $P(2,-1,0)$, $Q(4,1,1)$, and $R(4,-5,4)$.

(a) [1] Plot the points P, Q, and R.

note: $\vec{PQ} \cdot \vec{PR} = 0$
$\Rightarrow \vec{PQ} \perp \vec{PR}$
$\Rightarrow \triangle PQR$ is a right $\triangle$ with the $90°$ between $\vec{PQ}$ + $\vec{PR}$ ie at P



(+.5) started (b) [3] Find the length of $\overrightarrow{PQ}$

(+1) $\vec{PQ} = \langle 4-2, 1-1, 1-0 \rangle = \langle 2, 2, 1 \rangle$

(+1) so $\|\vec{PQ}\| = \sqrt{2^2+2^2+1} = \sqrt{4+4+1} = \sqrt{9} = 3$

(+.5) got it

(+.5) started (c) [3] Find the area of the triangle PQR.

(+1) recall that the area of the parallelogram with corners PQR is $\|\vec{PQ} \times \vec{PR}\|$

(+.5) $\vec{PR} = \langle 4-2, -5-1, 4-0 \rangle = \langle 2, -4, 4 \rangle$

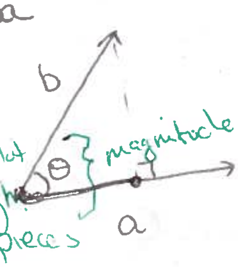
(+.5) so $\begin{Vmatrix} i & j & k \\ \vec{PR} & & \\ \vec{PQ} & & \end{Vmatrix} = \begin{vmatrix} i & j & k \\ 2 & 2 & 1 \\ 2 & -4 & 4 \end{vmatrix} = \vec{i}(8+4) - \vec{j}(8-2) + \vec{k}(-8-4) = 12\vec{i} - 6\vec{j} - 12\vec{k}$

(+.5) so $\frac{1}{2}$ · area of parallelogram $= \frac{1}{2}\|12\vec{i} - 6\vec{j} - 12\vec{k}\| = \frac{1}{2}\sqrt{144+36+144}$
$= \frac{1}{2}\sqrt{4(36+9+36)} = \sqrt{81} = 9$

3. [5] Find the projection of of the vector $\overrightarrow{b} = \langle 1, 1, 2 \rangle$ onto $\overrightarrow{a} = \langle -2, 3, 1 \rangle$.

Sohcahtoa

(+.5) start
(+.5) picture
(+1.5) angle/dot
(+.5) side length
(+1) plan/pieces



1) note $\|b\| = \sqrt{1^2+1^2+2^2} = \sqrt{6}$

2) $\cos\theta = \frac{\vec{b} \cdot \vec{a}}{\|\vec{b}\|\|\vec{a}\|} \Rightarrow \theta = \cos^{-1}\left(\frac{-2+3+2}{\sqrt{6}\sqrt{4+9+1}}\right) = \cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{14}}\right)$

now the magnitude is
$\cos\theta = \frac{\text{want}}{\|b\|} \Rightarrow \cos\left(\cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{14}}\right)\right) = \frac{\text{want}}{\|b\|}$

so $\frac{3}{\sqrt{6}\sqrt{14}} = \frac{\text{want}}{\|b\|} \Rightarrow \text{want} = \frac{3\|b\|}{\sqrt{6}\sqrt{14}} = \frac{3\sqrt{6}}{\sqrt{6}\sqrt{14}} = \frac{3}{\sqrt{14}}$

direction: same way as a so (+.5) $\frac{a}{\|a\|}$ {direction or $\frac{1}{\sqrt{4+9+1}}\langle -2, 3, 1 \rangle = \left\langle \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}} \right\rangle$

(+.5) put the 2 together: $\frac{3}{\sqrt{14}}\left\langle \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}} \right\rangle$ or $\left\langle \frac{-6}{14}, \frac{9}{14}, \frac{3}{14} \right\rangle = \left\langle \frac{-3}{7}, \frac{9}{14}, \frac{3}{14} \right\rangle$