TMATH 126: Quiz 2

Key

You may use any work of yours that you made from last week. This includes, practice problems from the book and worked out WebAssign problems. This *does not* include photocopies of notes from the book or tutorials shown on WebAssign. You may also use a calculator, but you are not allowed to use any device that can access the internet.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

   started (+.5) reason (+1) got it (+.5)

   T (F) An infinite sum of nonzero terms will never converge to a finite number.

   Geometric series with $|r|<1$ work

   ex $\frac{1}{2} + (\frac{1}{2})^2 + (\frac{1}{2})^3 + (\frac{1}{2})^4 + \ldots = 1$

   (T) F If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

   Consider the contrapositive: if $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$

   Certainly if you keep adding #s that aren't getting smaller - your infinite sum won't 'stop growing'

   T (F) If $\{a_n\}$ is a sequence such that $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

   harmonic series

   $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \ldots$

2. Consider the sequence: $\left\{3, \frac{3}{5}, \frac{3}{25}, \frac{3}{125}, \frac{3}{625} \ldots\right\}$.

   (a) [2] Find a formula for the n^{th} term where we start counting at one.

   $\frac{3}{5^{n-1}}$ works

   started (+.5) indexing (+1) form (+.5)

   (b) [1] Find the limit of the terms in the above sequence as $n \to \infty$.

   $\lim_{n\to\infty} \frac{3}{5^{n-1}} = 0$ den is going to ∞

   got (+1)

3. [6] Find the following limits (if they exit):

$$\lim_{n\to\infty} \sin\left(\frac{-n\pi}{6n+3}\right)$$

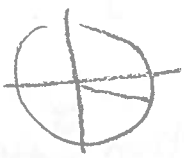
started (+.5) cont (+.5)

$= \sin\left(\lim_{n\to\infty} \frac{-n\pi}{6n+3}\right)$ "$\frac{\infty}{\infty}$"

limit laws (+.5)

$\overset{L'H}{=} \sin\left(\lim_{n\to\infty} \frac{-\pi}{6}\right)$ L'H (+1)

$= \sin\left(\frac{-\pi}{6}\right) = -\frac{1}{2}$ eval (+.5)



$$\sum_{n=1}^{\infty} \frac{3n^2-n}{n^2+4}$$

started (+.5) limit laws (+1.5)

note $\lim_{n\to\infty} \frac{3n^2-n}{n^2+4} = \lim_{n\to\infty} \frac{3 - \frac{1}{n}}{1+\frac{4}{n^2}} = 3$

reasoning (+1): If $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ diverges so this series diverges.

4. If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = n2^{-n} + 5$.

(a) [3] Find $\sum_{n=1}^{\infty} a_n$ if it exists. Justify your answer.

started (+.5) limit laws (+1) [illegible] (+1) understood (+1) reasoning (+.5)

$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n = \lim_{n\to\infty} (n2^{-n}+5) = \lim_{n\to\infty} \left(\frac{n}{2^n}+5\right)$

$= \lim_{n\to\infty} \left(\frac{n}{2^n}\right) + 5$ "$\frac{\infty}{\infty}$" $\overset{L'H}{=} \lim_{n\to\infty} \left(\frac{1}{2^n \ln 2}\right) + 5$

$= 0 + 5 = 5$

(b) [2] Find $\lim_{n\to\infty} a_n$ if it exists. Justify your answer.

started (+.5) reasoning (+1) got it (+.5)

Since $\sum_{n=1}^{\infty} a_n$ converged, we know $\lim_{n\to\infty} a_n = 0$ by true/false on opposite side.

or

started (+.5) reasoning for a_n (+.5) found a_n (+.5) took limit (+.5)

note $a_n = S_n - S_{n-1} = (n2^{-n}-5) - ((n+1)2^{-(n+1)} - 5) = \frac{n}{2^n} - \frac{n+1}{2^{n+1}}$

$= \frac{2n-(n+1)}{2^{n+1}} = \frac{n+1}{2^{n+1}}$ and $\lim_{n\to\infty} \frac{n+1}{2^{n+1}} =$ "$\frac{\infty}{\infty}$"

$\overset{L'H}{=} \lim_{n\to\infty} \frac{1}{2^{n+1}\ln 2}$

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