TMATH 126: Quiz 1

Key

This is an open note, open book, and open peer group quiz. You are welcome to use whatever materials you would like for this quiz except for people outside the class. For example, you may not ask me, the TLC student workers, or other faculty for help.

Groups of 3 or 4 are mandatory. Only one quiz should be turned in per group. If there is only one name on the top of a quiz that is turned in, I *will not* mark it.

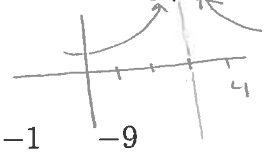
Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [3] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample. Let f and g be functions each with the domain $\mathbb{R}$.

T F If $\lim_{x\to 3} f(x) = 5$ and $\lim_{x\to 3} g(x) = 0$, then $\lim_{x\to 3} \dfrac{f(x)}{g(x)}$ does not exist.

note: $\frac{5}{(x-3)}$ has no limit (even ∞) as $x \to 3$

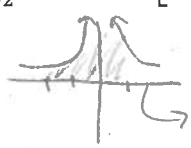
consider $\dfrac{5}{(x-3)^2}$



So $\lim_{x\to 3} \frac{5}{(x-3)^2} = \infty$

So could exist if ∞ exists.

T F $\displaystyle\int_{-2}^{1} x^{-4}\,dx = \left[\frac{-1}{3x^3}\right]_{-2}^{1} = \frac{-1}{3} - \frac{-1}{-24} = \frac{-9}{24}.$

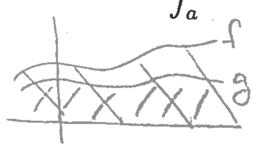


x^{-4} is not cont $[-2,1]$ so FTC won't work

area should be positive

T F If $g(x) < f(x)$ for $a \le x \le b$, then $\displaystyle\int_a^b g(x)\,dx \le \int_a^b f(x)\,dx$.

picture helps



Shaded area corresponds with $\int_a^b g(x)$ which is smaller than $\int_a^b f(x)\,dx$

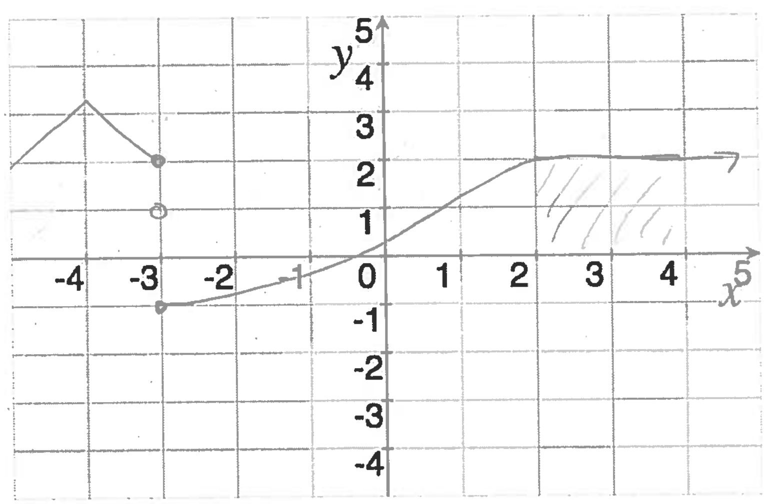
2. [5] Sketch the graph of an example function f that satisfies the following conditions: (+1)

(a) the domain of f is $\mathbb{R}$ (+1)

(b) $f'(-4)$ is not defined (+1)

(c) $\lim_{x\to -3} f(x) \neq f(-3)$ (+1)

(d) $\displaystyle\int_2^4 f(x)\,dx = 4$ (+1)



3. [4] Determine the following, if they exist:

$\lim_{x\to\infty} x\sin(\frac{3\pi}{x}) = \lim_{x\to\infty} \frac{\sin\frac{3\pi}{x}}{\frac{1}{x}}$ "$\frac{0}{0}$"

L'H (+.5) $= \lim_{x\to\infty} \frac{\frac{-3\pi}{x^2}\cos\frac{3\pi}{x}}{\frac{-1}{x^2}}$ (+.5) $= \lim_{x\to\infty} 3\pi\cos(\frac{3\pi}{x})$

$= 3\pi \lim_{x\to\infty} \cos(\frac{3\pi}{x}) \to \cos 0 = 1$ $= 3\pi \cdot 1$

(+.5) took limit/notation (+.5) $= 3\pi$

$\lim_{x\to\infty} e^{-x}\cos(x) = \lim_{x\to\infty} \frac{\cos x}{e^x}$

note $-1 \le \cos x \le 1$ (+.5)

$\Rightarrow \frac{-1}{e^x} \le \frac{\cos x}{e^x} \le \frac{1}{e^x}$ squeeze (+.5)

since $\lim_{x\to\infty} -1/e^x = 0 = \lim_{x\to\infty} 1/e^x$

took limit/notation (+.5)

by the squeeze thm (+.5)

$\lim_{x\to\infty} e^{-x}\cos(x) = 0$

4. [4] Let $f(x) = xe^{-3x}$.

(a) Find the equation of the line tangent to f when $x = 5$.

Looking for $y = mx + b$ (+.5)

$m = f'(5)$ finding f' first:

$f'(x) = xe^{-3x} \cdot -3 + e^{-3x}$

$= -3xe^{-3x} + e^{-3x}$

So $m = f'(5)$ (+.5) $= -15e^{-15} + e^{-15}$

$= \frac{-15}{e^{15}} + \frac{1}{e^{15}} = \frac{-14}{e^{15}}$

have: $y = -14e^{-15}x + b$

passes thru $(5, 5e^{-15})$ (+.5)

So (+.5)

$5e^{-15} = -14e^{-15} \cdot 5 + b$

$\Rightarrow b = 5e^{-14} + 70e^{-15}$

$\therefore y = (-14e^{-15})x + (5e^{-14} + 70e^{-15})$

(+1)

(b) Use the linear approximation you found in part (a), to estimate $f(5.2)$. $\approx -.00000428x + .00002$

$f(5.2) \approx -14e^{-15}(5.2) + (5e^{-14} + 70e^{-15})$

(+.5) computed (+.5)

$\approx 3.30117 \cdot 10^{-6}$

5. [4] Find the following:

$\int x\cos(2x)\,dx$

$u = x \quad v = \frac{1}{2}\sin(2x)$ (+1)

$du = dx \quad dv = \cos(2x)$

integration by parts: $uv - \int v\,du$ (+.5)

$\frac{1}{2}x\sin(2x) - \int \frac{1}{2}\sin(2x)\,dx$

$= \frac{1}{2}x\sin(2x) + \frac{1}{2}\frac{1}{2}\cos(2x) + C$

$\frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C$ got it (+.5)

Check $[\frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C]'$ ✓

notation (+.5)

$\int \frac{y + 6y^7}{y^2}\,dy = \int \frac{y}{y^2} + \frac{6y^7}{y^2}\,dy$

$= \int y^{-1} + 6y^5\,dy = \ln|y| + y^6 + C$ (+.5) (+.5)

abs val (+.5)

Check: $[\frac{-1}{y} + y^5 + C]'$ ✓

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