Exam 1

Tmath 126

Spring 2011

1. [] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

   (a) If there exists some number M such that $a_n \leq M$ for all n, then $\{a_n\}$ converges.

   (b) The Taylor series is an example of a power series.

   (c) Given a function f, the associated taylor series T has the property that $f(x) = T(x)$ for all x.

   (d) $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] Write the following sum in expanded form:

$$\sum_{n=1}^{4} \frac{\sqrt{2n+1}}{n!}$$

3. [4] Write the following sum using the sigma notation:

$$1 - \frac{2}{3} + \frac{3}{9} - \frac{4}{27} + \frac{5}{81}$$

4. [20] Compute the following if possible.

$$\lim_{n\to\infty} \frac{9^{n+1}}{10^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{2}{n}$$

5. Let $p(x) = x^6 - 5x^2 + 3x - 2$.

   (a) [5] Find the second order Taylor polynomial $T_2(x)$ based at $b = 1$.

   (b) [5] Bound the error $|p(x) - T_2(x)|$ on the interval [0.5,1.5].

6. [10] Find the Taylor series expansion for $\dfrac{x}{4+x}$ centered at 0, and find out where it converges.

7. [10] Compute the following indefinite integral.

$$\int \frac{\sin(x)}{x}\,dx$$

8. [10] Use geometric series to show $0.99999\ldots = 1$.