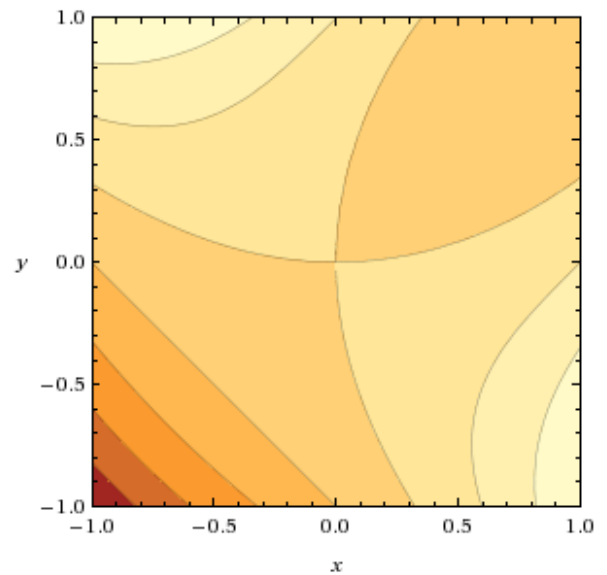


Optimizing with Constraints

1. Find all local extrema of the function $f(x, y) = 4 + x^3 + y^3 - 3xy$.

2. Maximize the the function $f(x, y) = 4 + x^3 + y^3 - 3xy$ when restricted to the disk $x^2 + y^2 \leq 1$. Note that the contours for f are provided to the right and can be used to verify your answer.

Contour plot:



3. Look again at the previous problem with a slightly more geometric approach.

(a) Draw the boundary to the disk $x^2 + y^2 = 1$ on the contour plot of f .

(b) Consider the point(s) where f is maximized on the curve of $x^2 + y^2 = 1$. Is there anything you notice about the contour lines of f and the curve $x^2 + y^2 = 1$?

(c) If you have the contour lines of a function, how can you determine the direction of the gradient vector?

(d) Do you notice anything about the gradient of f and the gradient of the restrictive circle $x^2 + y^2 = 1$?

Contour plot:

