Exam 2

Tmath 126

Spring 2011

1. [12] TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   Let $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ be vectors in $\mathbb{R}^3$.
   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   (a) $(\overrightarrow{a} \cdot \overrightarrow{b}) \cdot \overrightarrow{c} = \overrightarrow{a} \cdot (\overrightarrow{b} \cdot \overrightarrow{c})$.

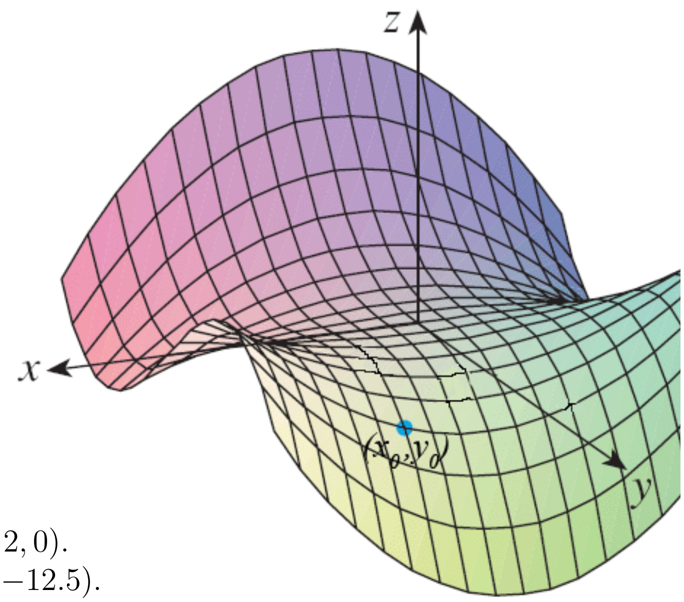
   (b) $||\overrightarrow{a} \times \overrightarrow{b}|| = ||\overrightarrow{b} \times \overrightarrow{a}||$.

   (c) If $f(x, y)$ is a continuous function, the first-order derivatives exist, and $f_x(0, 0) = 0 = f_y(0, 0)$, then f has a local minimum or maximum at the point $(0, 0)$.

   (d) If $f(x, y)$ is a continuous function, the first-order derivatives exist, and f has a local minimum or maximum at the point $(0, 0)$, then $f_x(0, 0) = 0 = f_y(0, 0)$.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider the function g pictured to the right and answer the following multiple choice questions.



(a) [1] $g_x(x_0, y_0)$ is

i. > 0

ii. < 0

(b) [1] $\left.\dfrac{\partial g}{\partial y}\right|_{(x_0,y_0)}$ is

i. > 0

ii. < 0

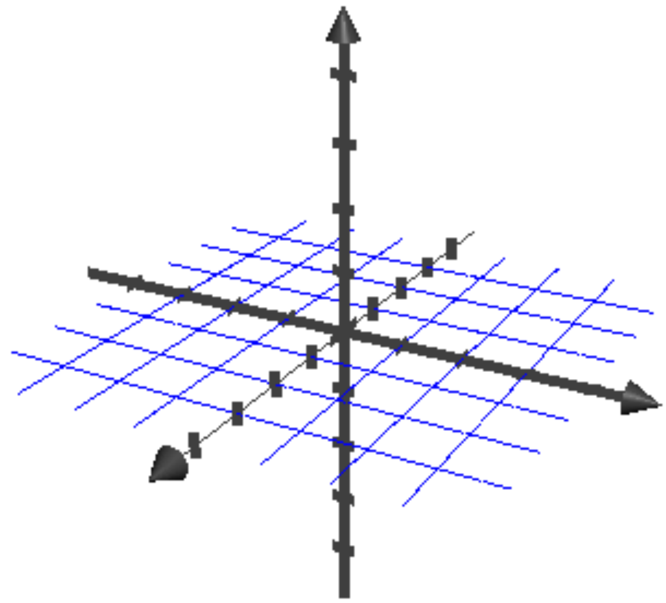
3. Let $P(1,3,2)$, $Q(3,-1,6)$, and $R(5,2,0)$. Also let $S(3,6,1.5)$ and $T(-9,-14,-12.5)$.

(a) [3] Find the equation of the line that passes through P, R, and Q.

(b) [2] Does the line that passes through S and T intersect the plane you found in part (a)? Justify your answer.

4. Consider the points: $P(1, -3, -2)$, $Q(2, 0, -4)$, and $R(6, -2, -5)$.

(a) [2] Plot the points P, Q, and R.



(b) [3] Find the length of $\overrightarrow{PR}$

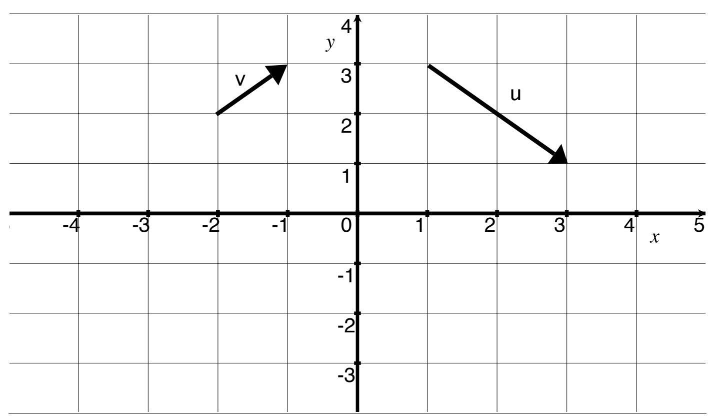
(c) [4] Use calculus methods to determine of $\triangle PQR$ is a right triangle or not.

5. Consider the vector $\overrightarrow{v}$ and $\overrightarrow{u}$ shown to the right.

(a) [1] Draw the vector $2\overrightarrow{u}$.

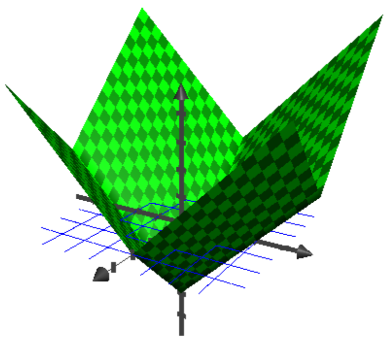
(b) [1] Draw the vector $-3\overrightarrow{v}$.

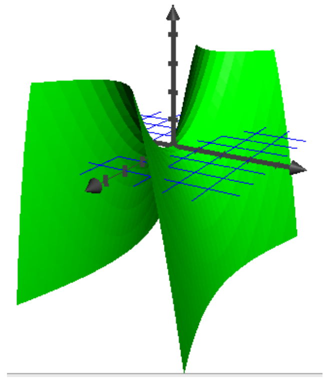
(c) [2] Draw the vector $2\overrightarrow{v} - \overrightarrow{u}$.

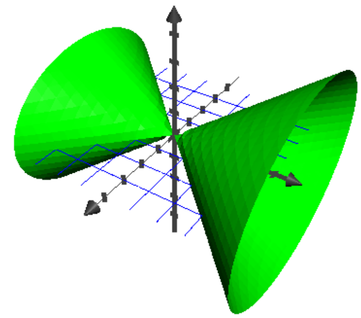


6. [3] Match the following equations to their respective graphs:

A. $z = \frac{x^2}{4} - y^2$ B. $z = |x| + |y| - 3$ C. $y^2 = x^2 + 2z^2$

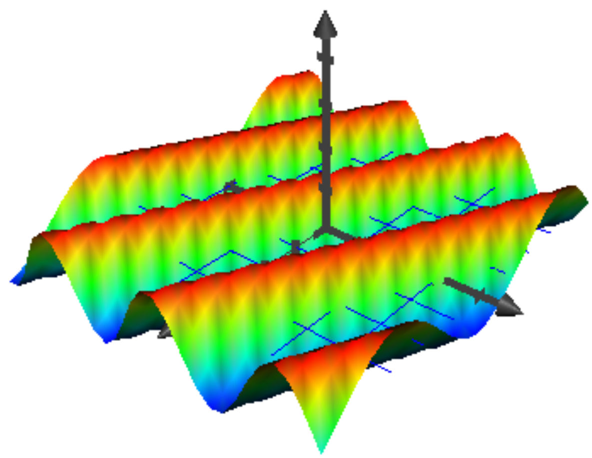






7. Consider the function $f(x, y) = -\sin(x + 2y)$ for the following questions.

(a) [3] Find the gradient of f .

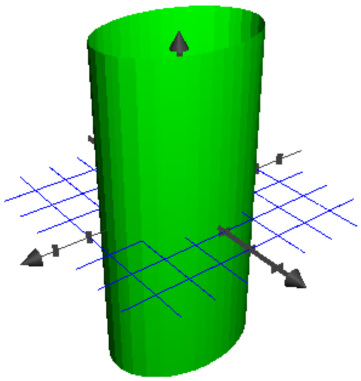
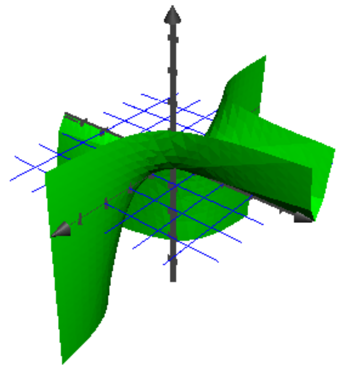
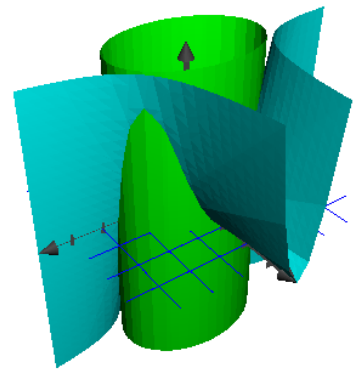


(b) [1] Evaluate the gradient at the point $(0, 0)$.

(c) [2] Interpret your answer in (b) graphically and consider referencing the graph of f shown to the right.

(d) [3] Find the linear approximation of f at the point $(0, 0)$.

8. Both the cylinder described by $6 = x^2 + 2y^2$ and the surface described by $z = x^2y$, along with their intersections are shown below.

(a) [6] Use Calculus methods to to find the (x, y, z) coordinates in $\mathbb{R}^3$ to find the points on both surfaces with the maximum z value.