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Exam 1 **Tmath 126** Key Spring 2011

Fix →

1. (6/~~8~~) TRUE/FALSE: ~~Circle T~~ in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, ~~circle F~~ and provide a counterexample.

(Quiz 2)

(a) An infinite sum of nonzero terms will never converge to a finite number.

False

started (+.5)
counterex (+1)
sense/notation (+.5)

ex any geometric series with a ratio r such that |r|<1 would work, like $\sum_{k=1}^{\infty} 1\left(\frac{1}{2}\right)^{k-1}$

(practice test #3)

(b) The series, $\sum_{n=1}^{\infty} \frac{2}{n}$ converges.

False $\sum_{n=1}^{\infty} \frac{2}{n} = 2\underbrace{\sum_{n=1}^{\infty} \frac{1}{n}}$ ↳ harmonic series which diverges.

(lecture 4/6)

(c) For all real numbers x, $e^{ix} = \cos(x) + i\sin(x)$, where $i = \sqrt{-1}$.

True Consider the series of $e^{ix} = 1 + i\frac{x}{1} - \frac{x^2}{2!} - i\frac{x^3}{3!} + \frac{x^4}{4!} + i\ldots$ along with the series of $\cos(x) + i\sin(x)$. They are equal.

(~~lec 4/11~~)

~~(d) In the real numbers, $.\overline{99} = 1$.~~

~~True If you expand~~ $.\overline{9}$ into a geometric series the first term is .9 + the ratio is 1/10. Since $|1/10| < 1$ the series converges to $\frac{.9}{1-1/10}$ or 1

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] Write the following sum using the sigma notation:

$$-\frac{1}{4} + \frac{2}{9} - \frac{3}{16} + \frac{4}{25}$$

(WebHW 2 #3)

$$\sum_{n=1}^{4} \frac{(-1)^n n}{(n+1)^2}$$

(+.5) (+.5) (+1)

started (+.5)

index (+1.5)

3. [8] Compute the following if possible.

$$\sum_{n=0}^{\infty} \frac{(2)^n}{n!}$$

(Practice exam #3 & webHW2#10)

Note $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

So $\sum_{n=0}^{\infty} \frac{(2)^n}{n!} = e^2$

alg manip (+.5)
started (+1)
connect (+1.5)
interpret 2 (+1)
~~[illegible]~~ (+X)

$$\sum_{n=1}^{\infty} \frac{(2e)^n}{6^{n-1}} = \sum_{n=1}^{\infty} 2e\left(\frac{2e}{6}\right)^{n-1} = 2e + 2e\frac{2e}{6} + 2\left(\frac{2e}{6}\right)^2 + \dots$$

started (+.5) alg (+.5) Ans

geometric series (+1)

$a = 2e$ (+.5)

$r = 2e/6 = e/3 < 1$ (+1)

Thus series converges to

(+.5) $\frac{2e}{1-e/3} = \frac{2e}{\frac{3-e}{3}} = \frac{6e}{3-e}$

4. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that the n^{th} partial sum of a series is $s_n = \dfrac{n-n^2}{3n^2-e}$.

(a) [3] Find $\sum_{n=1}^{\infty} a_n$, if it exists. Justify your answer.

(Quiz 2#4)
(+.5) started
(+1) Correct s_n to Σ
(+1) took limit
(+.5) got it

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{n-n^2}{3n^2-e} = \lim_{n\to\infty} \frac{\frac{n-n^2}{n^2}}{\frac{3n^2-e}{n^2}} = \lim_{n\to\infty} \frac{\frac{1}{n}-1}{3-\frac{e}{n^2}} = -\frac{1}{3}$$

So $-\frac{1}{3}$

(b) [3] Find ~~[illegible]~~ a_n.

(+.5) started
(+1) Correct s_n to a_n
(+.5) alg
(+1) reasoning/notation

~~$\frac{1-1^2}{3(1)^2-e}$~~

Note $a_n = (a_1+a_2+a_3+\dots+a_{n-1}+a_n) - (a_1+a_2+a_3+\dots+a_{n-1}) = s_n - s_{n-1}$

$$= \frac{n-n^2}{3n^2-e} - \frac{(n-1)-(n-1)^2}{3(n-1)^2-e} = \frac{n-n^2}{3n^2-e} - \frac{n-1-n^2+2n-1}{3(n-1)^2-e}$$

$n-2n+1$

(c) [2] Find $\lim_{n\to\infty} a_n$, if it exists. Justify your answer.

(+.5) started
(+.5) reasoning/logic
(+.5) got it
(+.5) limit

Since $\sum_{n=1}^{\infty} a_n$ converges to a finite number

it means $\lim_{n\to\infty} a_n = 0$

or

$$\lim_{n\to\infty}\left(\frac{n-n^2}{3n^2-e} - \frac{(n-1)-(n-1)^2}{3(n-1)^2-e}\right) = \lim_{n\to\infty}\left(\frac{\frac{1}{n}-1}{3-\frac{e}{n^2}} - \frac{\frac{1}{n}-\frac{1}{n^2}-1+\frac{2}{n}-\frac{1}{n^2}}{3-\frac{6}{n}+\frac{1}{n^2}+\frac{e}{n^2}}\right)$$

$$= \lim\left(-\frac{1}{3} - -\frac{1}{3}\right) = 0$$

5. [3] For what x values with the power series $\sum_{n=1}^{\infty}(x-4)^n$ converge?

~~Webtt4~~
lecture 4/4)

Geometric series that conv if (+1)

$|r| < 1$ note $r = (x-4)$

$\Rightarrow |x-4| < 1$ (+.5)

$-1 < x-4 < 1$ alg (+1)/ solve for x

$3 < x < 5$ notation (.5)

move R

6. Let $f(x) = \cancel{\frac{\ln(3x)}{3x}}$. $\ln(3x)$

practice exam #5)

(a) 6 [5] Find the second order Taylor polynomial $T_2(x)$ based at $b = \frac{1}{3}$.

k	$f(x)$	$f(\frac{1}{3})$
0	$\ln(3x)$	0
1	$\frac{1}{3x}\cdot 3 = \frac{1}{x}$ .5	3 .5
2	$\frac{-1}{x^2}$ .5	-9 .5
	(+1)	(+1)

$f(\tfrac{1}{3}) + \frac{f'(\frac{1}{3})}{1!}(x-\tfrac{1}{3}) + \frac{f''(\frac{1}{3})}{2!}(x-\tfrac{1}{3})^2$ (+1)

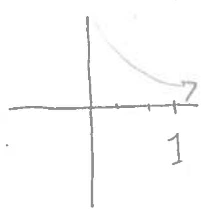
$\overset{.5}{0} + \overset{+1}{3(x-\tfrac{1}{3})} - \overset{+1}{\tfrac{9}{2}(x-\tfrac{1}{3})^2}$

plug in right (+.25)

notation +.5

(b) [4] Bound the error $|f(x) - T_2(x)|$ on the interval $\left[\frac{1}{6}, \frac{1}{2}\right]$.

3 $\quad \frac{2}{x^3}$ (+.5)



note $f^{(3)}(x)$ is a dec function finding M (+1.5)

$\therefore$ bounded by $f^{(3)}(\tfrac{1}{6}) = \frac{2}{(\frac{1}{6})^3} = 2\cdot 6^3$

(webhw4 #1-3)

error $\leq \frac{2\cdot 6^3}{3!}(x-\tfrac{1}{3})^3 \leq \frac{2\cdot 6^3}{3!}\left(\tfrac{1}{6}\right)^3 = \frac{2\cdot 6\cdot 6\cdot 6}{6\cdot 6\cdot 6\cdot 6} = \frac{1}{3}$

(+1) $\qquad$ (+.5)

for all x

steps (+.5)

3
36
6
216
+ 2

7. Consider the function e^{-x^2}.

got it (+.5)
start (+.5)
use this (+1)

if start +.5
if start dec. +.5
get a few dec +.5
plug in accurate +.5
use Taylor's right +1

(a) [3] Find the Taylor series for the function e^{-x^2}.

recall $e^u = 1 + \frac{u}{1!} + \frac{u^2}{2!} + \frac{u^3}{3!} + \frac{u^4}{4!} + \cdots$

(lecture 4/11)

$\Rightarrow \quad e^{-x^2} = 1 + \frac{-x^2}{1!} + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \frac{(-x^2)^4}{4!} + \cdots$ (+1)

$= 1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

(b) 3[2] Find $\int_0^x e^{-t^2}\, dt$.

$\int_0^x e^{-t^2}\, dt = \int_0^x 1 - \frac{t^2}{1!} + \frac{t^4}{2!} - \frac{t^6}{3!} + \frac{t^8}{4!} - \cdots \, dt$ sub (+.5)

int (+1.5)

$= \left[t - \frac{t^3}{3\cdot 1!} + \frac{t^5}{5\cdot 2!} - \frac{t^7}{7\cdot 3!} + \frac{t^9}{9\cdot 4!} - \cdots \right]_0^x$ plug in ends (+1)

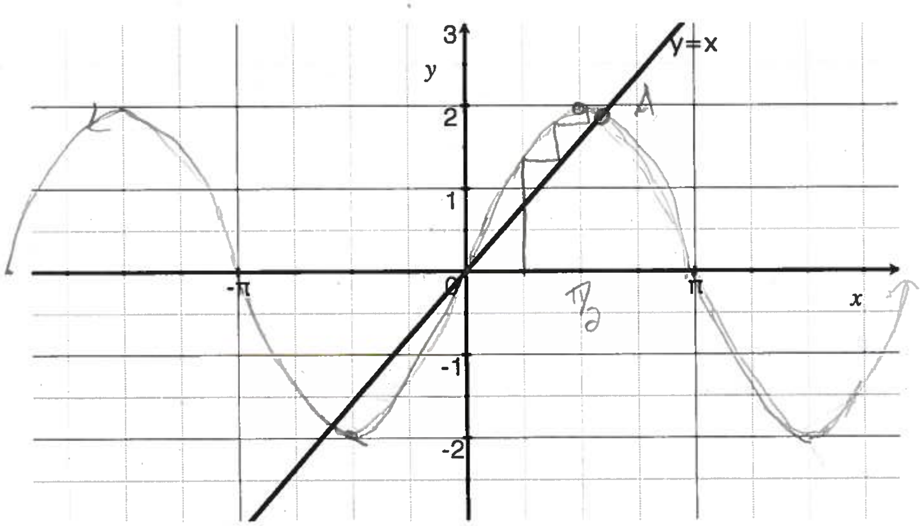
$= x - \frac{x^3}{3\cdot 1!} + \frac{x^5}{5\cdot 2!} - \frac{x^7}{7\cdot 3!} + \frac{x^9}{9\cdot 4!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)\, n!}$

8. Consider the sequence: $\{a_1, 2\sin(a_1), 2\sin(2\sin(a_1)), 2\sin(2\sin(2\sin(a_1))), \ldots\}$

(a) [1] The sequence above is an iterative sequence where $a_n = f(a_{n-1})$. Find $f(x)$.

$2\sin(x)$

(lecture 4/11)



(b) [1] Draw the graph of f on the axes provided.

(c) [1] If $a_1 = \frac{\pi}{4}$, identify what the sequence above converges to on the graph.

Converges to the y-value of the coor. A

(d) [2] For what values of a_1 will the above sequence converge to a negative finite number?

between -2π and π

Fix →

$(-\pi, 0)$

started +.5

(+.5) (+.5) (+.5) endpts.

52