Exam 1

Tmath 126

Spring 2011

1. [8] TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   (a) An infinite sum of nonzero terms will never converge to a finite number.

   (b) The series, $\sum_{n=1}^{\infty} \frac{2}{n}$ converges.

   (c) For all real numbers x, $e^{ix} = \cos(x) + i\sin(x)$, where $i = \sqrt{-1}$.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] Write the following sum using the sigma notation:

$$-\frac{1}{4} + \frac{2}{9} - \frac{3}{16} + \frac{4}{25}$$

3. [8] Compute the following if possible.

$$\sum_{n=0}^{\infty} \frac{(2)^n}{n!} \qquad\qquad \sum_{n=1}^{\infty} \frac{(2e)^n}{6^{n-1}}$$

4. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that the n^{th} partial sum of a series is $s_n = \dfrac{n-n^2}{3n^2-e}$.

   (a) [3] Find $\displaystyle\sum_{n=1}^{\infty} a_n$, if it exists. Justify your answer.

   (b) [3] Find a_n. You do *not* need to simplify the expression.

   (c) [2] Find $\displaystyle\lim_{n\to\infty} a_n$, if it exists. Justify your answer.

5. [3] For what x values with the power series $\sum_{n=1}^{\infty}(x-4)^n$ converge?

6. Let $f(x) = \ln(3x)$.

(a) [6] Find the second order Taylor polynomial $T_2(x)$ based at $b = \frac{1}{3}$.

(b) [4] Bound the error $|f(x) - T_2(x)|$ on the interval $\left[\frac{1}{6}, \frac{1}{2}\right]$.

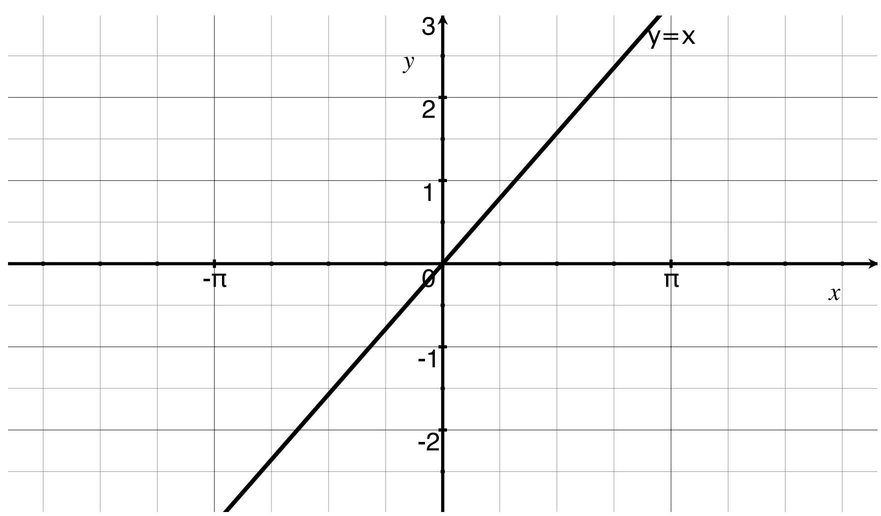
7. Consider the function e^{-x^2}.

(a) [3] Find the Taylor series for the function e^{-x^2}.

(b) [3] Find $\int_0^x e^{-t^2}\, dt$.

8. Consider the sequence: $\{a_1, 2\sin(a_1), 2\sin(2\sin(a_1)), 2\sin(2\sin(2\sin(a_1))), ...\}$

(a) [1] The sequence above is an iterative sequence where $a_n = f(a_{n-1})$. Find $f(x)$.



(b) [1] Draw the graph of f on the axes provided.

(c) [1] If $a_1 = \frac{\pi}{4}$, identify what the sequence above converges to on the graph.

(d) [2] For what values of a_1 between -2π and π will the above sequence converge to a negative finite number?