Median 7/26

Key

Exam 2 TMath 126 Autumn 2024

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) and a one-sided 8.5 by 11 sheet of notes. Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. [6] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

   (a) If $f_x(2,3) = 0$ then $(2,3,f(2,3))$ will be a critical point.

   False (+.5), we need $f_x(2,3) = 0$ AND $f_y(2,3) = 0$

   Start (+.5)
   Both partials = 0 (+1)
   sense (+.5)

   Know what critical point is (+.5)

   (b) To optimize the function $f(x,y) = e^{xy}$ subject to the constraint $x^3 + y^3 = 16$ we would need to solve the following system of equations:

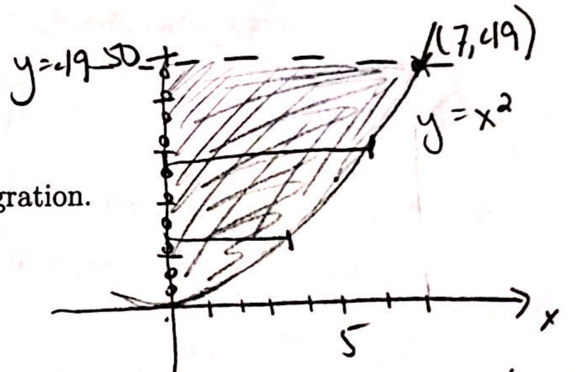
   (+.5) False, we are missing the LaGrange multipliers!

   $$\left\{\begin{array}{c} ye^{xy} = 3x^2 \\ xe^{xy} = 3y^2 \\ x^3 + y^3 = 16 \end{array}\right\} \tag{1}$$

   $\nabla f = \lambda \nabla C$
   where $C(x,y) = x^3 + y^3$

   Start (+.5)
   recognize Lagrange mult (+1)
   sense (+.5)
   notation (+.5)

2. Consider $\displaystyle\int_0^7 \int_{x^2}^{49} 1 + xy \, dy \, dx$



   (a) [2] (WebHW15.2 #7) Sketch the region of integration.

   $y = x^2$ $x = 0$
   $y = 49$ $x = 7$

   parabola (+.5)
   got it (+1)

   (+.5) read y boundaries as y boundaries + x boundaries as x boundaries

   (b) [2] (Practice Exam2 #4) Switch the order of integration.

   $$\int_0^{49} \int_{0}^{\text{parabola}} (1 + xy) \, dx \, dy \quad (\text{fnx } y)$$

   (+.5) if $y = x^2$
   $\pm\sqrt{y} = x$
   want
   $\sqrt{y} = x$
   b/c in quadrant 1

   $$\int_0^{49} \int_0^{\sqrt{y}} (1 + xy) \, dx \, dy$$

   (+.5) (+.5)

   notation (+.5)

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3. Consider the table that provides the heat index (I) as a function of temperature (T) and relative humidity (H).

Table 1 Heat Index I As a Function of Temperature and Humidity

Actual temperature (°F) T \ Relative humidity (%) H	50	55	60	65	70	75	80	85	90
90	96	98	100	103	106	109	112	115	119
92	100	103	105	108	112	115	119	123	128
94	104	107	111	114	118	122	127	132	137
96	109	113	116	121	125	130	135	141	146
98	114	118	123	127	133	138	144	150	157
100	119	124	129	135	141	147	154	161	168

(a) [2] (WrittenHW 14.3#2)

Estimate $\frac{\partial I}{\partial H}|_{(92,55)}$

(+.5)

$\approx \frac{\Delta I}{\Delta H} = \frac{105-100}{60-50} = \frac{5}{10} = \frac{1}{2}$

fix T (+.5)

vary H around 55 (+.5)

got it (+.5)

(b) [3] (Quiz6#2)

Find the linear approximation/linearization of I when $T = 92$ and $H = 55$.

(+.5) Looking for a tangent plane $z - z_0 = m_x(x-x_0) + m_y(y-y_0)$

$m_T = \frac{\partial I}{\partial T} \approx \frac{\Delta I}{\Delta T}$ (+.5) from (c)

$I - I_0 = m_T(T-T_0) + m_H(H-H_0)$

(+1) $\approx \frac{107-98}{94-90} = \frac{9}{4}$ $\quad m_H \approx \frac{1}{2}$ (+.5)

$I - 103 = \frac{9}{4}(T-92) + \frac{1}{2}(H-55)$

plug in correctly (+.5)

(c) [2] (WrittenHW 14.4#28) Use your linear approximation to approximate I when $T = 94$ and $H = 60$. Is the approximation an overestimate or an underestimate to the actual value?

Approx gives $I - 103 = \frac{9}{4}(94-92) + \frac{1}{2}(60-55)$

$I \approx 110$

plug in (+1)

note the actual Heat index @ $T=94$ + $H=60$

is 111 (+.5) graph reading

=> our approx is an underestimate. (+.5)

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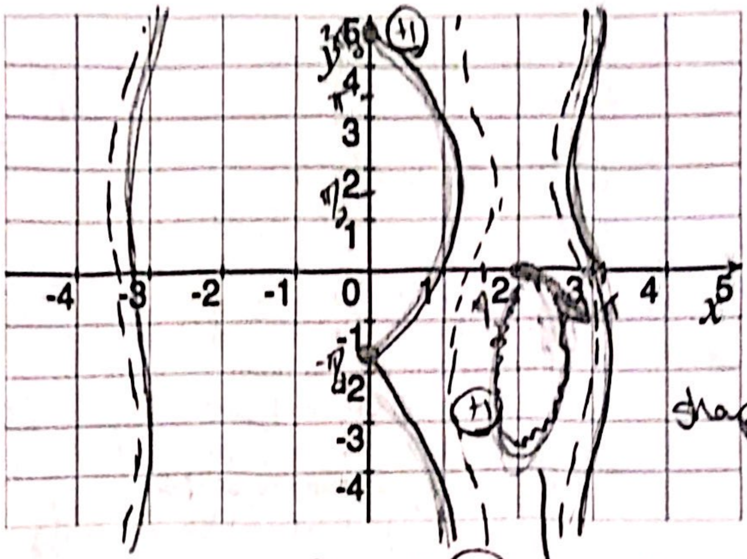
4. Let $f(x, y) = x^2 \sin(x) - \sin(y)$.

(a) [3] (Quiz5 #2) Draw sections of the contour map/the level curves of f when $z = 1$ and $z = 2$. Label the curves!



$1 = x^2\sin(x) - \sin(y)$

$1 + \sin(y) = x^2\sin(x)$

$2 = x^2\sin(x) - \sin(y)$

$2 + \sin y = x^2 \sin x$

set z to fixed # (+.5)

set to 1 and 2 (+.5)

if z = 4

shape +.5

(b) [3] (WebHW14.3) Find $f_x(x, y)$

$f_x(x,y) = x^2 \frac{\partial}{\partial x}(\sin(x)) + \frac{\partial}{\partial x}(x^2)\sin(x) - \frac{\partial}{\partial x}(\sin(y))$

$= x^2\cos(x) + 2x\sin(x) - 0$

(+.5)

trig derivative (+.5)

product Rule (+1)

wrt. x (+.5)

notation (+.5)

(c) [2] (WrittenHW14.6 #26) Sketch the direction of $\nabla f(2, 0)$ on the graph.

start @ (2,0) (+1)

directional ascent (+1)

Algebraically

$\langle 2^2\cos(2) + 2\cdot 2\sin(2), -\cos(0)\rangle$

or

$\langle 1.97, -1\rangle$

(note $f_y(x,y) = -\cos(y)$)

or graphically towards steepest ascent

can see easier if graph level curve when z=4

(d) [3] (DirectionActivity #2) Find $D_{\vec{u}}(2, 0)$ where $\vec{u} = \langle \frac{\sqrt{3}}{2}, \frac{1}{2}\rangle$.

(+.5) $D_{\vec{u}}(2,0) = \nabla f(2,0) \cdot \langle \frac{\sqrt{3}}{2}, \frac{1}{2}\rangle$

(already a unit vector)

note $\|\langle \frac{\sqrt{3}}{2}, \frac{1}{2}\rangle\| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1$

$= \langle x^2\cos(x) + 2x\sin(x), -\cos(y)\rangle\Big|_{(2,0)} \cdot \langle \frac{\sqrt{3}}{2}, \frac{1}{2}\rangle$

(+.5)

start +.5

plug in (+1)

$= \langle 4\cos(2) + 4\sin(2), -\cos(0)\rangle \cdot \langle \frac{\sqrt{3}}{2}, \frac{1}{2}\rangle$

$= \langle 1.97, -1\rangle \cdot \langle \frac{\sqrt{3}}{2}, \frac{1}{2}\rangle$

dot product (+1)

$= 1.97 \cdot \frac{\sqrt{3}}{2} + (-1)\frac{1}{2} \approx 1.2$

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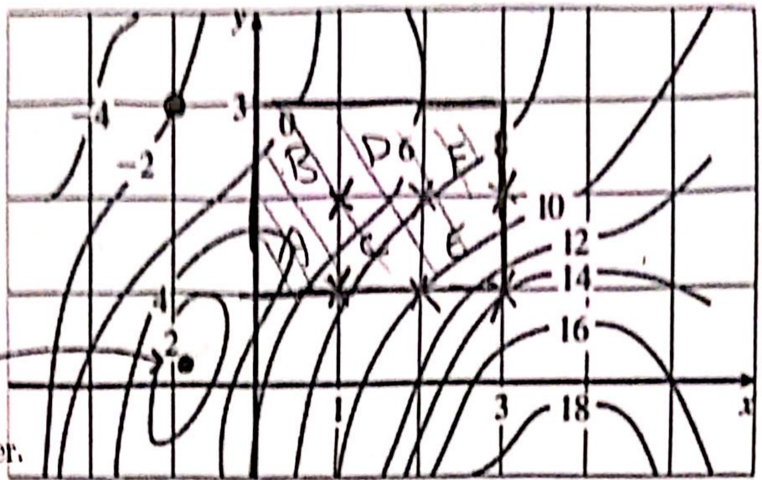
5. Let f have the contour lines shown on the right.



(a) [1] Estimate $f(-1,3)$

-2 plot point (+.5) got it (+.5)

(b) [2] (OptimizationActivity#1) Identify one critical point on the graph of f and identify it as a local minimum, maximum or neither.

$\approx(-.9, .2)$ (+1) looks like min (+1)

(c) [3] (PracticeExam2 #2) Estimate the volume bounded by f above the rectangle $0 \le x \le 3$ and $1 \le y \le 3$. Be clear about what choices you are making to estimate the volume. We'll estimate using six prisms where $\Delta x = 1$ (+.5) and $\Delta y = 1$ (+.5)

(1) region on graph

We'll add the volumes, let h_I be the height of prism I (+.5)

(.5) # of boxes move score

(.5) height explained

~~graph~~

$$[\ A + B + C + D + E + F$$

$$\Delta x\Delta y h_A + \Delta x\Delta y h_B + \Delta x\Delta y h_C + \Delta x\Delta y h_D + \Delta x\Delta y h_E + \Delta x\Delta y h_F$$

$$1\cdot1\cdot7 + 1\cdot1\cdot5 + 1\cdot1\cdot10 + 1\cdot1\cdot8 + 1\cdot1\cdot14 + 1\cdot1\cdot9 = 53$$

[We choose the lower, right corner of each square

6. [6] (§14.7 ex5) Find the shortest distance between the point $(1, 0, -2)$ and the surface described by $x + 2y + z = 4$. Note that this problem can be solved in dramatically different ways!!! If you choose to solve this using chapter 14 techniques you can outline the solution making sure to include:

Start (.5)

(a) (.5) definitions of variables used,

(b) identifying the function that needs to be optimized,

(c) boxing systems of equations that need to be solved (but do not solve them!), &

(d) explaining how you would verify your work is correct (ie a maximum)

§14.7 techniques

Dist from $(1,0,-2)$ to points on $z = 4-x-2y$

$$= D(x,y,z) = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \quad (+1)$$

$$= \sqrt{(x-1)^2 + (y-0)^2 + (z+2)^2}$$

$$= \sqrt{(x-1)^2 + y^2 + (4-x-2y+2)^2}$$

To minimize we can just focus on minimizing the function below the sq (·).

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(+1) $f(x,y) = (x-1)^2 + y^2 + (4-x-2y+2)^2$

↳ function to be optimized

(+1) System of equations

$$\begin{cases} f_x(x,y)=0 \\ f_y(x,y)=0 \end{cases} \text{ or } \boxed{\begin{cases} 2(x-1)+2(6-x-2y)(-1)=0 \\ 2y+2(6-x-2y)(-2)=0 \end{cases}}$$

took derivatives correctly (+1)

Once I have the critical point I should be able to use the second derivative test. ... but I'd probably just look at the graph (+1)