Quiz 5

Key

This is a two-stage quiz.During the first stage, use your knowledge & calculator to take this quiz. You have 15 min. In the second stage. You are now welcome to use your books, notes, and others in the class to retake the same quiz in the remaining half hour. One copy from each group of 2 to 3 people must be turned in.

Show *all* your work. Reasonable supporting work must be shown for any partial credit.

1. Given that g is a differentiable function with $g(6,3) = 5$, $g_x(6,3) = 1$, and $g_y(6,3) = -1$.

   (a) [3] Find the local linearization of g when $x = 6$ and $y = 3$.

   3D Tangent Plane Activity #2 — start (+.5); eq of plane (+.5); plug in slopes/f_x/f_y (+1); plug in points (+1)

   $L(x,y) - z_0 = m_x(x - x_0) + m_y(y - y_0)$

   $L(x,y) - 5 = 1(x-6) - 1(y-3)$

   $L(x,y) = 5 + x - 6 - y + 3$ (linearization) OR $L(x,y) = x - y + 2$

   (b) [1] Approximate $g(6.2, 2.9)$ using tangent planes. — Webwork 14.4 #3

   plug in $x = 6.2$ (+.5), $y = 2.9$ (+.5)

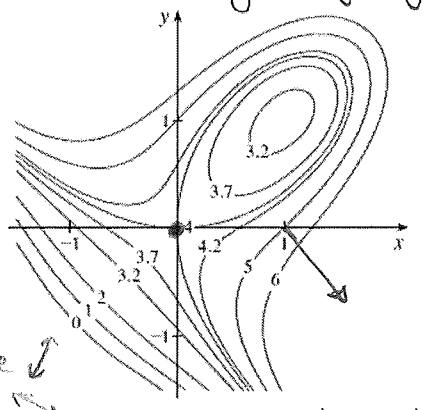
   $g(6.2, 2.9) \approx 5 + (6.2 - 6) - (2.9 - 3)$

   $= 5 + .2 + .1 = 5.3$

2. Consider the function $f(x,y) = 4 + x^3 + y^3 - 3xy$ whose contour curves are shown below.

   $f(x,y) = 4 + x^3 + y^3 - 3xy$

   $f_x(x,y) = 3x^2 - 3y$ $\quad f_y(x,y) = 3y^2 - 3x$

   (a) [2] Sketch the gradient vector at the point $(1,0)$. — Webwork 14.6 #6

   vector (+1)

   increasing $z = f(x,y)$ values (+1)

   note: we can do this exactly ☺

   $\langle f_x(1,0), f_y(1,0) \rangle = \langle 3, -3 \rangle$

   (b) [1] Identify a critical point on the graph of f that is not a local minimum or maximum. — Optimization Activity #1

   give point (+.5) / def of CP — only need one value

   $(0,0)$ works — note decr when move ↙ inc when move ↘ — get it (+.5)

   (c) [3] Find the directional derivative of f at $(1,0)$ in the direction of $\vec{u} = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$ — Webwork 14.6

   directional derivative

   (+.5) $= \nabla f \cdot \vec{u}$

   $= \langle f_x, f_y \rangle \cdot \vec{u}$ (+.5) (+.5)

   $= \langle 3x^2 - 3y, 3y^2 - 3x \rangle \cdot \langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$

   $= \frac{1}{\sqrt{2}}(3x^2 - 3y) - \frac{1}{\sqrt{2}}(3y^2 - 3x)$ (+.5)

   note

   $\|\vec{u}\| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{-1}{\sqrt{2}}\right)^2}$

   $= \sqrt{\frac{1}{2} + \frac{1}{2}}$

   $= \sqrt{1} = 1$

   so unit vector?

   evaluating at $(1,0)$: (+.5)

   $\frac{1}{\sqrt{2}}(3 \cdot 1^2 - 3 \cdot 0) - \frac{1}{\sqrt{2}}(3 \cdot 0^2 - 3(1))$

   $= \frac{3}{\sqrt{2}} + \frac{3}{\sqrt{2}} = \frac{6}{\sqrt{2}} = \frac{3 \cdot 2}{\sqrt{2}}$

   $= 3\sqrt{2}$

   looks to agree w/ (b) — got it (+.5)