Key

Quiz 4

This is a two-stage quiz. During the first stage, use your knowledge & calculator. You have 15 min. In the second stage, you are now welcome to use your books, notes, and students in the class to retake the same quiz. You have the remainder of the quiz time to write one solution (with everyone's name on it!!!) to be turned in for the group.

should not have told you this?

Show *all* your work. Reasonable supporting work must be shown for any partial credit.

1. [4] (TrigActivity#4) Describe the strategy for evaluating $\int \cot^m(x)\csc^n(x)\,dx$ when n is even. Consider the two worked out examples below.

Start (+.5) generalizing (+.5)

(+1) Use substitution with $u = \cot(x)$

(+1) Reserve a $\csc^2 x$ for the du

(+1) Take any other powers of $\csc x$ (which must be even) can be removed with $\csc^2 x = 1 + \cot^2(x)$

$\int \cot(x)\csc^2(x)\,dx$

$u = \cot(x)$

$du = (\cot(x))' = \left(\frac{\cos x}{\sin x}\right)'$

$= \frac{-\sin x \sin y - \cos(y)\cos(x)}{\sin^2 x}\,dx$

$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}\,dx$

$= \frac{-1}{\sin^2 x}\,dx \; \boxed{= -\csc^2 x\,dx}$

so

$\int \cot(x)\csc^2(x)\,dx = \int u(-1)\,du$

$= -\frac{1}{2}u^2 + C$

$= -\frac{1}{2}\cot^2(x) + C$

CK: $\left[-\frac{1}{2}\cot^2(x) + C\right]' = \frac{1}{2}\cdot 2\cot(x)\csc(x)$ ✓

$\int \cot^2(x)\csc^4(x)\,dx$

$= \int u^2\csc^2(x)\csc^2(x)\,dx$

$= \int u^2 \csc^2(x)(-du)$

$= -\int u^2(1 + \cot^2(x))\,du$

$= -\int u^2(1+u^2)\,du$

$= -\int u^2 + u^4\,du$

$= -\frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$= -\frac{1}{3}\cot^3(x) - \frac{1}{5}\cot^5(x) + C$

$u = \cot(x)$

$du = -\csc^2 x\,dx$

$\boxed{-du = \csc^2 x\,dx}$

recall

$\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

CK: $\left[-\frac{1}{3}\cot^3(x) - \frac{1}{5}\cot^5(x) + C\right]'$

$+\frac{1}{3}\cdot 3\cot^2(x)\csc^2(x) + \frac{1}{5}\cdot 5\cot^4(x)\csc^2(x) + 0$

$= \cot^2(x)\csc^2(x)(1 + \cot^2(x))$

$= \cot^2(x)\csc^2(x)\csc^2(x)$ ✓

2. [3] (WebHW7-1) Given that $f(1) = 4$, $f(4) = 5$, $f'(1) = 7$, $f'(4) = 5$ and that f'' is continuous. Find $\int_1^4 2xf''(x)\,dx$.

try method correctly (+1.5)

notation (+.5)

note - you do not need to do completely to get all points?

$u = 2x \qquad v = f'(x)$

$du = 2dx \qquad dv = f''(x)dx$

$\int 2x\,f''(x)dx = \int u\,dv = uv - \int v\,du$

$2xf'(x) - \int f'(x)\,2\,dx$

$2xf'(x) - 2f(x)$

So $\int_1^4 2xf''(x)\,dx$

$= \left[2xf'(x) - 2f(x)\right]_1^4$

$[2(4)f'(4) - 2f(4)] - [2\cdot 1 f'(1) - 2f(1)]$ (+1) plug in $f'(c)$ & $f''(c)$

$(8\cdot 5 - 2\cdot 5) - (2\cdot 7 - 2\cdot 4) = (40 - 10) - (14 - 8) = 30 - 6$

$= 24$

3. A particle is moving along a straight line and has a velocity $v(t) = te^t$ meters per second after t seconds.

   (a) [1] Find the velocity when $t = 2$.

   $v(2) = 2e^2 \approx 14.7$ m/s

   (b) [2] (WrittenHW7.1#75) Find the expression that you could give to technology that would return the change in distance in the first 2 seconds.

integral (+.5)

bounds (+.5)

got it (+.5)

function (+.5)

$\int_0^2 te^t\,dt = 8.4$

(with tech)

OR

$\int te^t\,dt = te^t - \int e^t\,dt = te^t - e^t + C$

$u = t \qquad v = e^t$

$du = dt \qquad dv = e^t\,dt$

$\int_0^2 te^t\,dt = \left[te^t - e^t\right]_0^2$

$= (2e^2 - e^2) - (0 - e^0)$

≈ 8.4