Exam 1

TMath 126

Practice

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

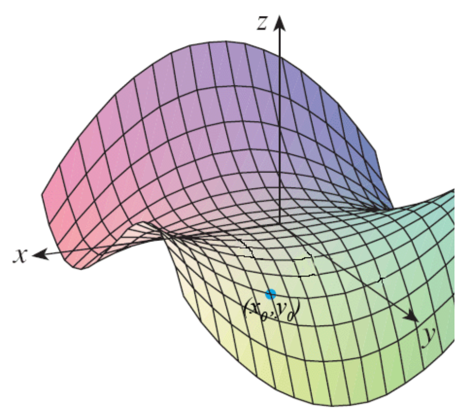
   Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be vectors in $\mathbb{R}^3$.
   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   (a) If $\vec{u} \cdot \vec{v} = 0$, then $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$.

   (b) $(\vec{u} \times \vec{w}) \cdot \vec{w} = 0$

   (c) $\dfrac{\vec{u} \cdot \vec{v}}{||\vec{u}||||\vec{v}||} = \dfrac{\vec{u}}{||\vec{u}||} \cdot \dfrac{\vec{v}}{||\vec{v}||}$.

   (d) The line $(2+3t, -4t, 5+t)$ where $t \in \mathbb{R}$ intersects the point $(-4, 8, 3)$.

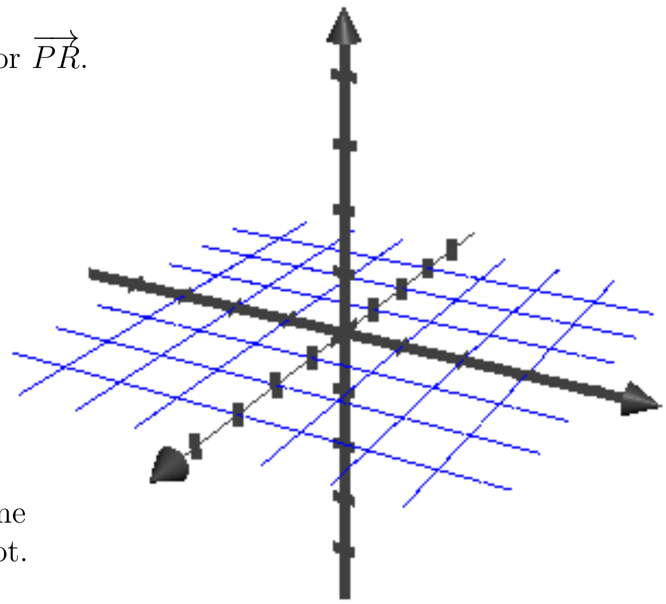
   

   (e) Consider the function g pictured to the right.
   $g_x(x_0, y_0) > 0$.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider the points: $P(1,3,2)$, $Q(3,-1,6)$, and $R(5,2,0)$. Also let $S(3,6,1.5)$ and $T(-9,-14,-12.5)$.

   (a) Plot the points P, Q, and R.

   (b) Find the components of the vector $\overrightarrow{PR}$.

   (c) Find the length of $\overrightarrow{PR}$.

   (d) Draw the vector $\overrightarrow{PR} - 2\,\overrightarrow{j}$ and then write its components.

   (e) Use calculus methods to determine if $\triangle PQR$ is a right triangle or not.

   (f) Find the equation of the plane that passes through P, R, and Q.

   (g) Does the line that passes through S and T intersect the plane you found in part (a)? Justify yourself.

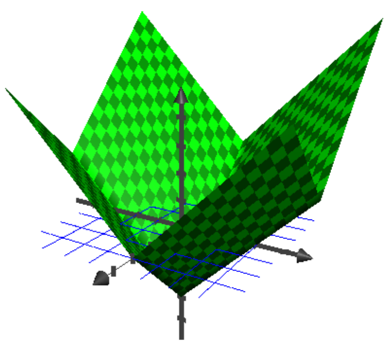
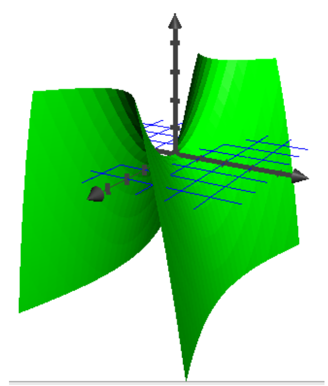
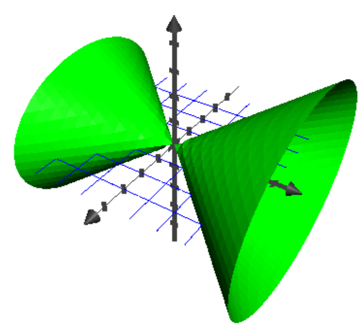
3. [3] Consider the equation $2z = \dfrac{x^2}{2} - 2y^2$.

   (a) Does the above equation describe a function of x and y? Why or why not?

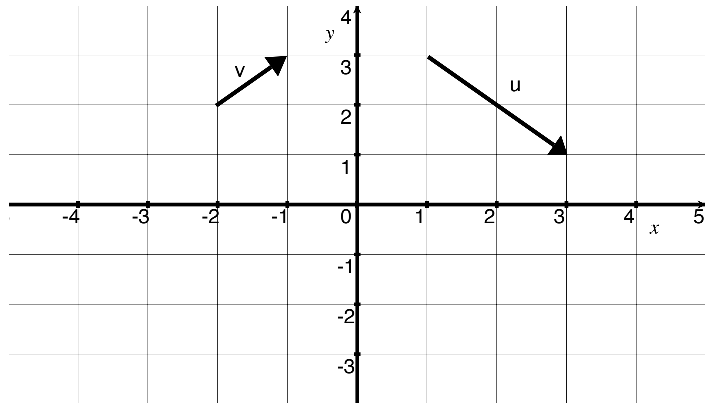
   (b) Describe the contour curves of the graph of the equation above. That is, describe the intersection of the graph of the above equation with the planes $z = k$ where k is some constant.

   (c) Describe the intersection of the graph of $2z = \dfrac{x^2}{2} - 2y^2$ with planes parallel to the xz axis. That is, when $y = k$ for some constant k.

   (d) Which (if any) of the following is a graph of the above function?

4. Consider the vector $\overrightarrow{v}$ and $\overrightarrow{u}$ shown to the right.



   (a) Draw the vector $-3\overrightarrow{v}$.

   (b) Draw the vector $2\overrightarrow{v} - \overrightarrow{u}$.

   (c) Find the projection of $\overrightarrow{u}$ onto $\overrightarrow{v}$.

5. We define $\overrightarrow{r}(t)$ by: $x(t) = 1 + t^4$, $y(t) = te^{-t}$, and $z(t) = \sin(2t)$.

   (a) Find the line tangent to the curve $\overrightarrow{r}(t)$ when $t = 0$.

   (b) Find the length of the arc traced by $\overrightarrow{r}(t)$ from $t = 0$ to $t = 5$.

6. In a study of frost penetration it was found that the temperature T at time t (measured in days) at a depth x (measured in feet) can be modeled by the function

$$T(x,t) = T_0 + T_1 e^{-\lambda x} \sin(\omega t - \lambda x)$$

where $\omega = 2\pi/360$ and λ is a positive constant.

(a) Find $\frac{\partial T}{\partial x}$. What is its physical significance?

(b) Find $\frac{\partial T}{\partial t}$. What is its physical significance?