Ave 73%
Median 75%

Key

Exam 2 TMath 126 ~~40 points~~ Autumn 2023

1. [12] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

   (a) (WebHW14.4#2) The equation of the plane tangent to $z = \ln(x-8y)$ at $(9,1,0)$ is $z = \underbrace{\frac{1}{x-8y}(x-9) + \frac{-8}{x-8y}(x-1)}$.

   also this should be y?

   start justify (1.5)
   sense (1.5)
   eq of plane (1.5)
   got it (1.5)

   False (1.5)

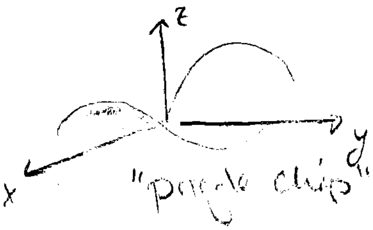
   not the equation of a plane?
   If $f(x,y) = \ln(x-8y)$, it looks like we found f_x and f_y but we forgot to plug in $(9,1,0)$ to get the slopes at the point. So should be $z = \frac{1}{9-8}(x-9) + \frac{-8}{9-8}(y-1)$.

   (b) (OptimizingActivity#1) If f is a function so that $f_x(3,-2) = 0$ and $f_y(3,-2) = 0$, then $f(3,-2)$ is a maximum or a minimum.

   start justify (1.5)
   sense (1.5)
   min/max/CP (1)
   got it (1.5)

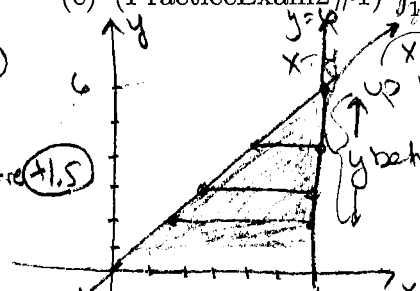
   False (1.5)

   The graph could have a saddle point (a 3D version of $y = x^3$ @ $(0,0)$)

   (c) (PracticeExam2#1) $\int_1^6 \int_y^6 x^2 \sin(x-y)\,dx\,dy = \int_0^6 \int_1^x x^2 \sin(x-y)\,dy\,dx$

   start justify (1.5)
   sense (1.5)
   areas in xy plane (1.5)

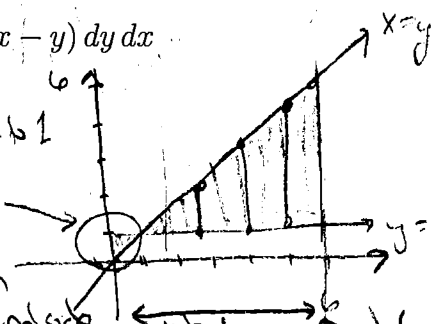
   x varies up to 6 and down to y
   y between 1 and 6

   False (1.5)

   Note: technology could have been used here?

   y varies up to x and down to 1

   Extra bit taken into account on the right hand side

   y between 0 and 6

   (d) (WebHW14.7#5) To find three positive numbers (x, y and z) whose sum is 160 but with a maximal product, we would want take the partial derivatives of the function $f(x,y,z) = x+y+z$ and set those equal to zero to find our critical points.

   start justify (1.5)
   sense (1.5)
   how to maximize (1)
   got it (1.5)

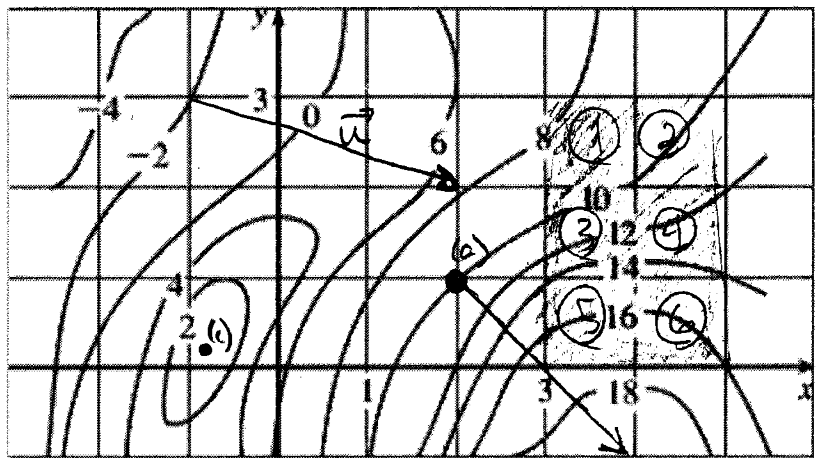
   Want to maximize $x \cdot y \cdot z$ with the constraint $x+y+z=160$.
   We want to find the CP of the function we want to maximize that is $f(x,y,z)$ should be $x \cdot y \cdot z$.

   False (1.5)

12

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Let f have the contour lines shown on the right.



(a) [1] Estimate $f(2,1)$

10

(b) [2] (§14.6 #26)Sketch the direction of the vector $\nabla f(2,1)$ on the graph.

⊥ to contour line (+.5)

(+.5) direction of steepest increase

vector (+.5) @ (2,1) (+.5)

(probably pretty long like 5 or 6 units but we only need the direction)

(c) [2] (quiz5#2) Identify one critical point on the graph of f and identify it as a local minimum, maximum or neither.

$\approx (-.8, .2)$ (+.5) local minimum (+1)

Know CP (+.5)

(d) [3] (3DCalculusActivity#4) Let $\vec{u} = \langle 3, -1\rangle$ Determine whether the directional derivative of f at point $(-1,3)$ along $\vec{u}$ is positive, negative, or zero. *Justify* your answer.

(+.5) Positive (+.5)

Sort justify (+.5)

At (-1,3) the z coord is -2. As we move along $\vec{u}$ (+.5) the z values are increasing to 0 and 6. So going up ? (+1) positive change in z :)

(e) [3] (IntegrationActivity#1) Estimate the volume bounded by f above the rectangle $3 \le x \le 5$ and $0 \le y \le 3$. Be clear about what choices you are making to estimate the volume.

(There are lots of ways to do this!)

(+.5) shaded region (+1) graph reading

(+.5) [I'll break the region up into 6 square pieces for the estimate so

Partial +.5 integrand

height$_1$ · Area$_1$ + height$_2$ · Area$_2$ + ... + height$_6$ · Area 6

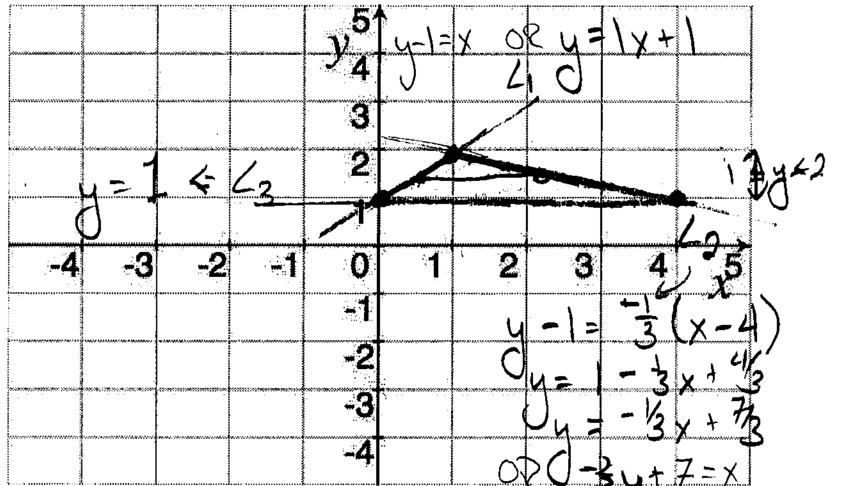
(+.5) [The squares are all 1 by 1 => Area$_i$ = 1·1 for i=1,...,6

(+.5) [To decide the height I'll estimate the height in the center of each square so

9·1 + 10·1 + 12·1 + 13·1 + 16·1 + 16·1 = 76 so ≈ 76 units^3

11

3. (WebHW15.2 #4) Consider the integral $\iint_D 2y^2 dA$ where D is the triangular region with the vertices $(0,1)$, $(1,2)$, and $(4,1)$



(a) [2] Draw the region D on the provided axis to the right.

points (+1.5)
triangle (+.5)

(b) [4] Express the double integral as an iterated integral. (ie figure out the bounds so that technology can compute this for you.)

finding equations of lines (+2)

$$\int_1^2 \int_{L_1}^{L_2} 2y^2 \, dx\, dy$$

bounds match dx/dy (+.5)
bounds (+1.5)
integrand (+.5)

$$\int_1^2 \int_{y-1}^{7-3y} 2y^2 \, dx\, dy$$

OR

$$\int_0^1 \int_{L_3}^{L_1} 2y^2 \, dy\, dx + \int_1^4 \int_{L_3}^{L_2} 2y^2 \, dy\, dx$$

$$= \int_0^1 \int_1^{x+1} 2y^2 \, dy\, dx + \int_1^4 \int_1^{-\frac{1}{3}x+\frac{7}{3}} 2y^2 \, dy\, dx$$

4. [4] (§14.4 #28) A function f of two variables is known to be continuous and provide the values in specified to the right.

$y \setminus x$	1.0	1.1	1.2
2.0	5	7	10
2.2	4	6	8
2.4	3	5	6

Your boss would like you to develop a linear model that could be used to estimate the value of $f(.8, 2.35)$. Build the model and justify the choices/steps that you make.

justify (1.5)

(1.5) [looking for $z - z_0 = m_x(x-x_0) + m_y(y-y_0)$

(1.5) [I'll try to do my computations around (1.0, 2.2) b/c that's pretty close to (.8, 2.35).

(+1) [$m_x = \frac{\Delta z}{\Delta x} = \frac{6-4}{1.1-1.0} = \frac{2}{.1} = 20$ (as y is held constant @ 2.2)

(+1) [$m_y = \frac{\Delta z}{\Delta y} = \frac{3-4}{2.4-2.2} = \frac{-1}{.2} = -5$ (as x is held constant @ 1.0)

so

$$z - 4 = 20(x-1.0) - 5(y-2.2)$$

plug in (+.5)

10

28
17
40

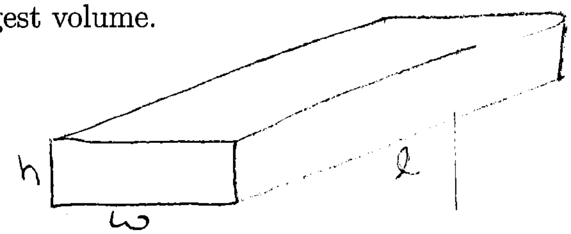
5. [7] (§14.8 #54) For the following problem you will outline (not actually find!) a solution. Make sure your outline includes:

   (a) definitions of variables used,
   (b) identifying the function that needs to be optimized,
   (c) boxing systems of equations that need to be solved (but do not solve them!), &
   (d) explaining how you would verify your work is correct (ie a maximum)

   A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its length and girth (the perimeter of a cross-sectional perpendicular to the length) is at most 108 inches. We want to find the dimensions of the package that can be mailed with the largest volume.

Start (+.5)
notation (+.5)

(a) box dimensions (+1)



(b) optimize volume so $f(h,w,l) = h \cdot w \cdot l$ (+1)

(c) use constraint
girth + length = 108
$2h+2w+l = 108$
so $r(h,w,l) = 2h+2w+l$ (+.5)

(+.5) Lagrange multipliers?
$\left\{ \begin{array}{l} \nabla f = \mu \nabla r \\ r(h,w,l) = 108 \end{array} \right\}$ (+.5)
⇓

$$\boxed{\begin{array}{l} wl = \mu 2 \\ hl = \mu 2 \\ hw = \mu 1 \\ 2h+2w+l = 108 \end{array}}$$

derivatives (+1.5)

OR (c) use constraint
girth + length = 108
$2h+2w+l = 108$
$\Rightarrow l = 108 - 2h + 2w$ (+.5)
Sub into $f(h,w,l)$ so
sub (+.5)
$f(h,w) = hw(108 - 2h + 2w)$
$= 108hw - 2h^2w + 2hw^2$
find critical points so
$\left\{ \begin{array}{l} f_w = 0 \\ f_h = 0 \end{array} \right\}$ (+.5)
⇓

$$\boxed{\begin{array}{l} 108h - 2h^2 + 4hw = 0 \\ 108w - 4hw + 2w^2 = 0 \end{array}}$$

derivatives (+1.5)

(d) Too many variables for a second derivative test..

Use $2h+2w+l = 108$
or $l = 108 - 2h + 2w$ and
sub into $f(h,w,l)$ so
only 2 variables?
Then graph. ↓

method that works (+1)

7

(d) Second derivative might work here
$4h \cdot (-4h) - (108 - 4h + 4w)^2$
evaluated at CP's & 2nd der test
(+1) Otherwise we can simply use graphing technology