Exam 2 **TMath 126** Autumn 2023

1. [12] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

   (a) (WebHW14.4#2) The equation of the plane tangent to $z = \ln(x - 8y)$ at $(9, 1, 0)$ is $z = \frac{1}{x-8y}(x - 9) + \frac{-8}{x-8y}(x - 1)$.

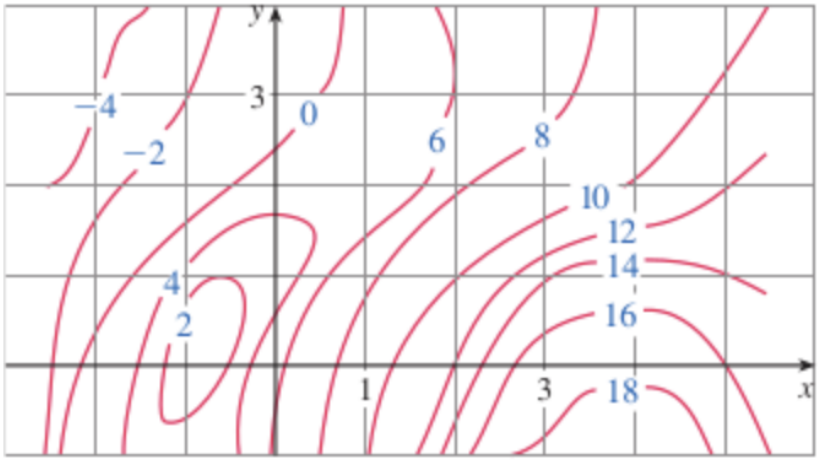
   (b) (OptimizingActivity#1) If f is a function so that $f_x(3, -2) = 0$ and $f_y(3, -2) = 0$, then $f(3, -2)$ is a maximum or a minimum.

   (c) (PracticeExam2#1) $\int_1^6 \int_y^6 x^2 \sin(x - y)\, dx\, dy = \int_0^6 \int_1^x x^2 \sin(x - y)\, dy\, dx$

   (d) (WebHW14.7#5) To find three positive numbers (x, y and z) whose sum is 160 but with a maximal product, we would want take the partial derivatives of the function $f(x, y, z) = x + y + z$ and set those equal to zero to find our critical points.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Let f have the contour lines shown on the right.



(a) [1] Estimate $f(2,1)$

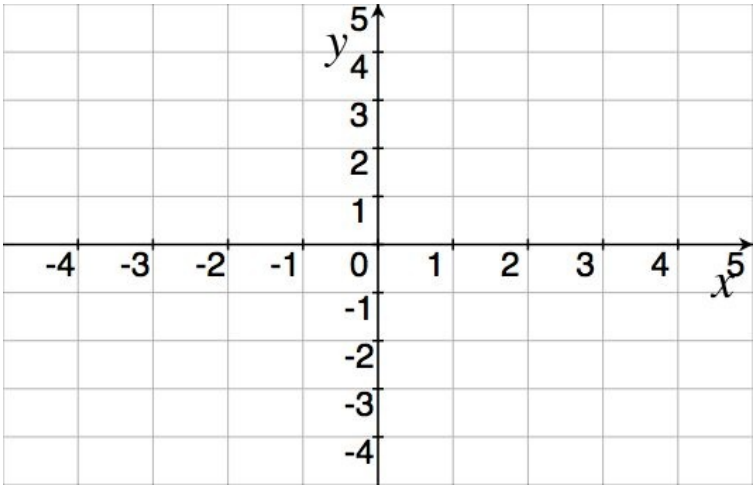
(b) [2] (§14.6 #26)Sketch the direction of the vector $\nabla f(2,1)$ on the graph.

(c) [2] (quiz5#2) Identify one critical point on the graph of f and identify it as a local minimum, maximum or neither.

(d) [3] (3DCalculusActivity#4) Let $\vec{u} = \langle 3, -1 \rangle$ Determine whether the directional derivative of f at point $(-1, 3)$ along $\vec{u}$ is positive, negative, or zero. *Justify* your answer.

(e) [3] (IntegrationActivity#1) Estimate the volume bounded by f above the rectangle $3 \leq x \leq 5$ and $0 \leq y \leq 3$. Be clear about what choices you are making to estimate the volume.

3. (WebHW15.2 #4) Consider the integral $\iint_D 2y^2 dA$ where D is the triangular region with the vertices $(0, 1)$, $(1, 2)$, and $(4, 1)$



(a) [2] Draw the region D on the provided axis to the right.

(b) [4] Express the double integral as an iterated integral. (ie figure out the bounds so that technology can compute this for you.)

4. [4] (§14.4 #28) A function f of two variables is known to be continuous and provide the values in specified to the right.

$y \setminus x$	1.0	1.1	1.2
2.0	5	7	10
2.2	4	6	8
2.4	3	5	6

Your boss would like you to develop a linear model that could be used to estimate the value of $f(.8, 2.35)$. Build the model and justify the choices/steps that you make.

5. [7] (§14.8 #54) For the following problem you will outline (not actually find!) a solution. Make sure your outline includes:

   (a) definitions of variables used,

   (b) identifying the function that needs to be optimized,

   (c) boxing systems of equations that need to be solved (but do not solve them!), &

   (d) explaining how you would verify your work is correct (ie a maximum)

   A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its length and girth (the perimeter of a cross-sectional perpendicular to the length) is at most 108 inches. We want to find the dimensions of the package that can be mailed with the largest volume.