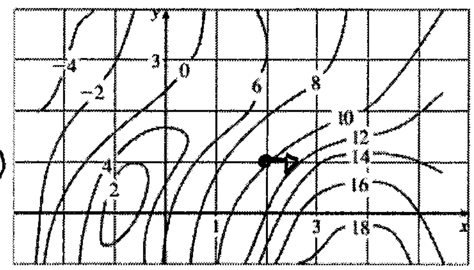
Median 68%
Average 63%

Key

Exam 1 TMath 126 Autumn 2023

1. [12] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

   (a) (Suggested §14.3#6) A contour map is given for a function f below. This map implies $f_x(2,1) \approx -2$.

   as x inc from 2 to 3 and y=1
   the z coord moves from 10 to 12
   so +2

   $\Rightarrow f_x(2,1) > 0$ False (+.5)

   stat. justify (+.5)
   sense (+.5)
   use partial der (+1)
   got it (+.5)

   (b) (dotActivity#1) If $\vec{w}$ and $\vec{v}$ are vectors in 3D, then $(\vec{w} \cdot \vec{j}) + \vec{v}$ returns a vector.

   False (+.5)

   recall $\vec{j} = \langle 0,1,0 \rangle$ and $\vec{w} \cdot \vec{j} = 0w_1 + 1w_2 + 0w_3 = w_2$
   which is a scalar.
   We have no way of adding scalars to vectors so
   $(\vec{w} \cdot \vec{j}) + \vec{v}$ = scalar + vector makes no sense $\therefore$ not vector

   stat justify (+.5)
   dot/vector add (+1)
   sense (+.5)
   got it (+.5)

   (c) (WebHW14.2#2) The limit $\displaystyle\lim_{(x,y)\to(\frac{3\pi}{2},\pi)} y\sin(x-y) = \pi$

   True (+.5)

   Well, sin is a cont function
   products + differences of cont functions are cont
   so... let's plug in $(3\pi/2, \pi)$...
   $\pi \sin\left(\frac{3\pi}{2} - \pi\right) = \pi \sin\left(\frac{\pi}{2}\right) = \pi \cdot 1 = \pi$

   stat justify (+.5)
   plug in/limit law/der (+1)
   sense (+.5)
   got it (+.5)

   (d) (§13.2#26) If $\vec{r}(t) = \langle 2^t, \ln(t+1), t \rangle$, then the line tangent to $\vec{r}(0)$ is:

   $$\langle 1,0,0 \rangle + \langle 2^t \ln(2), \frac{1}{1+t}, 1 \rangle$$

   does not define a line?

   False (.5)

   so, no, can't be a tangent line

   stat justify (+.5)
   line/tang (+1) der is vector #
   sense (+.5)
   got it (+.5)

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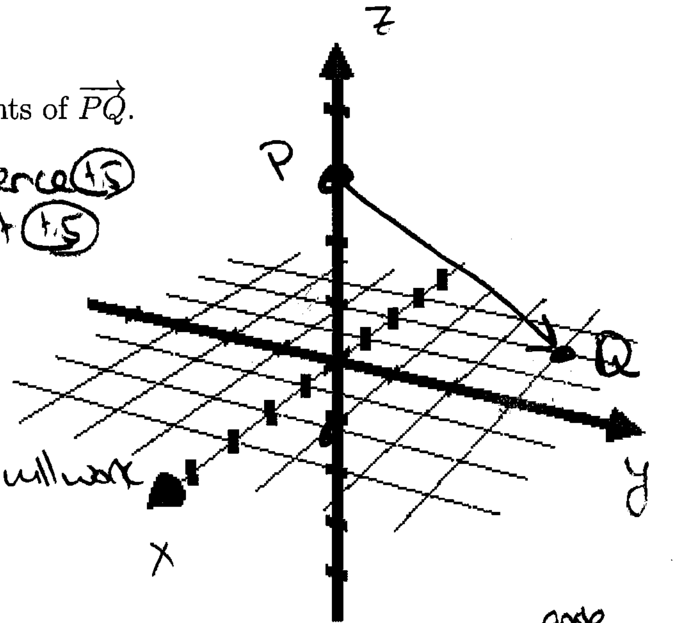
Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider the points $P(0,0,3)$ and $Q(-2,3,0)$



(a) [1] (PracticeExam1#2) Find the components of $\overrightarrow{PQ}$.

$\langle -2-0, 3-0, 0-3 \rangle$ difference (.5)

$= \langle -2, 3, -3 \rangle$ got it (.5)

(b) [2] (DotActivity#2) Find a vector parallel to $\overrightarrow{PQ}$.

vector (.5)
found one (1)
notation (.5)

any non-zero $t \cdot \langle -2,3,-3 \rangle$ will work

eg $t=1$ so $\langle 2,-3,3 \rangle$

$t=2$ $\langle -4,6,-6 \rangle$

(c) [3] (Quiz2#1) Find the angle $\overrightarrow{PQ}$ makes with $\langle 0,1,3 \rangle$.

angle between

dot/cross def (.5)
use correctly (+1.5)
solved for θ (+1)

Recall $\overrightarrow{PQ} \cdot \langle 0,1,3 \rangle = \|\overrightarrow{PQ}\| \cdot \|\langle 0,1,3 \rangle\| \cos\theta$

$\langle -2,3,-3 \rangle \cdot \langle 0,1,3 \rangle = \sqrt{4+9+9}\sqrt{0+1+9}\cos\theta$

$-2\cdot 0 + 3\cdot 1 + (-3)(3) = \sqrt{22}\sqrt{10}\cos\theta$

$\dfrac{-6}{\sqrt{22}\sqrt{10}} = \cos\theta \implies \theta = \arccos\left(\dfrac{-6}{\sqrt{22}\sqrt{10}}\right) \approx 1.99\text{rad}$ $113.9°$

(d) [3] (WebHW12.5 #4) Find an equation of a plane passing through $(2,1,0)$ and normal/orthogonal/perpendicular to $\overrightarrow{PQ}$

start (.5)
eq of plane (.5)
used correctly (+1.5)
got it (.5)

let's use $\vec{n} \cdot (\langle x,y,z \rangle - \langle x_0,y_0,z_0 \rangle) = 0$

orthogonal   point on plane

$\langle -2,3,-3 \rangle \cdot (\langle x,y,z \rangle - \langle 2,1,0 \rangle) = 0$

or

$-2(x-2) + 3(y-1) - 3(z-0) = 0$

$-2x + 4 + 3y - 3 - 3z = 0$

$-2x + 3y - 3z = -1$

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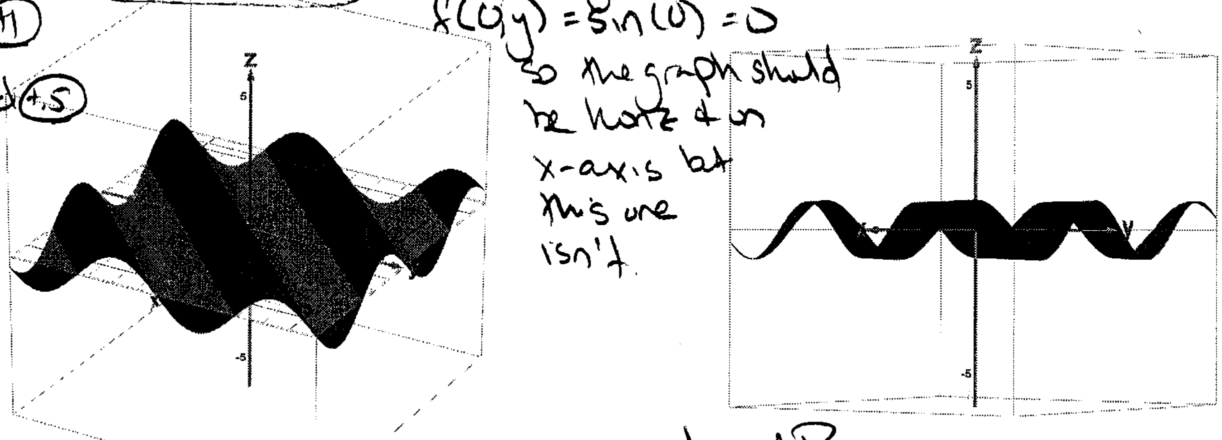
3. [3] (§14.1#64) Two perspectives of the graph of $f(x, y)$ are shown below. Identify which algebraic rule below corresponds with it. Provide justification!!!

sufficient (+.5)
reason for 1 (+.5)
reason for 2nd (+.5)

- ~~$f(x, y) = \sin(x) - \sin(y)$~~ ← this varies between -2 & 2 eg $f(\frac{\pi}{2}, -\frac{\pi}{2}) = 2$ but our graph stays between $-1 \le z \le 1$
- ~~$f(x, y) = \sin(xy)$~~ ← if $x = 0$ and y varies $f(0, y) = \sin(0) = 0$ so the graph should be horiz & on x-axis but this one isn't.
- $f(x, y) = \sin(x - y)$

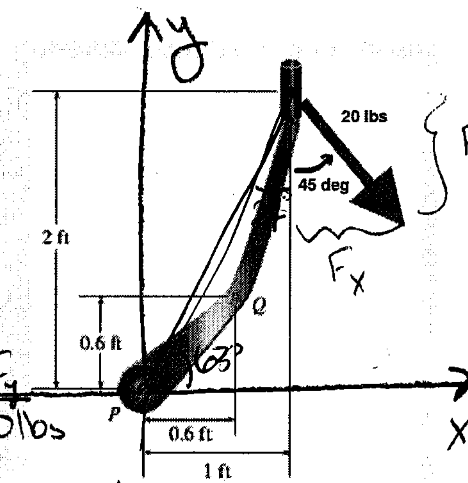
some analysis (+1)
circled/identified (+.5)



4. Consider the bicycle pedal shown on the right. A ~~horizontal~~ diagonal? force of 20 lbs is applied to the handle as shown.

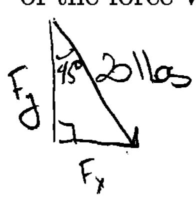
(a) [2] (3DActivity #1) Identify a 3D axis on the picture indicating the positive x, y, and z axis.

put x, y, z on (+1) right hand rule (+1)



(b) [3] (WrittenHW12.4#40) Write the components of the force vector with respect to your 3D axis.

wrt axis (+1)
Sohcahtoa correctly (+1.5)
alg (+.5)



Sohcahtoa

$\sin 45° = \frac{F_x}{20\text{lbs}}$ $\left(\cos 45° = \frac{F_y}{20\text{lbs}}\right.$

$\Rightarrow F_x = 20 \cdot \sin 45° = 20 \cdot \frac{1}{\sqrt{2}}$ $\Rightarrow F_y = 20 \cdot \frac{1}{\sqrt{2}}$

So $\langle \frac{20}{\sqrt{2}}, -\frac{20}{\sqrt{2}}, 0 \rangle$ or $\langle 10\sqrt{2}, -10\sqrt{2}, 0 \rangle$ or $(14.1, 14.1, 0)$

(c) [3] (Quiz2#2) Find the *vector* of the torque created about the pivot point P.

(+.5) recall $\vec{\tau} = \vec{r} \times \vec{F}$ $\quad \vec{r} = \langle 1, 2, 0 \rangle$

$\vec{r}$ vector (+1)
cross calculation (+1.5)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 10\sqrt{2} & -10\sqrt{2} & 0 \end{vmatrix} = \vec{i}(2 \cdot 0 - 0 \cdot 10\sqrt{2}) - \vec{j}(1 \cdot 0 - 0 \cdot 10\sqrt{2}) + \vec{k}(-10\sqrt{2} - 2 \cdot 10\sqrt{2})$$

$$= \vec{k}(-10\sqrt{2} - 20\sqrt{2}) = -30\sqrt{2}\,\vec{k} \approx -42.4\vec{k}$$

so $30\sqrt{2}$ into the page

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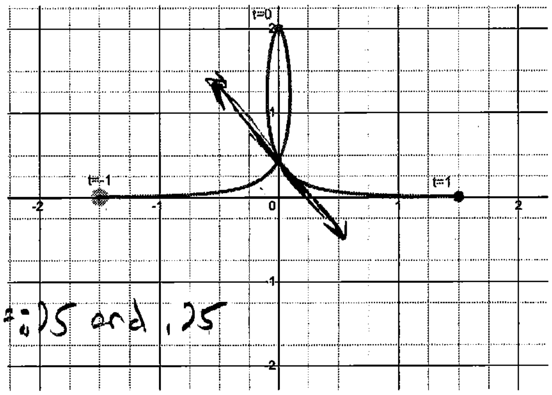
5. Consider the parametric curve $x = f(t)$, $y = g(t)$ where $-1 \leq t \leq 1$, graphed below for the following questions.



(a) [3] Looking at the graph, approximate where $\frac{dy}{dx}$ is not defined. (Report either a point on the graph or an approximate t value.)

start (+.5)

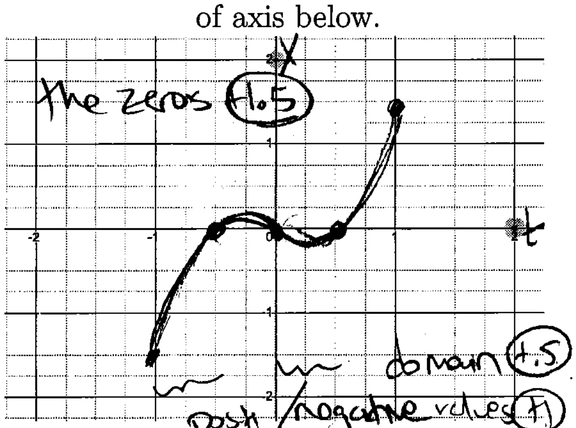
(+1) → @ vertical tangent lines

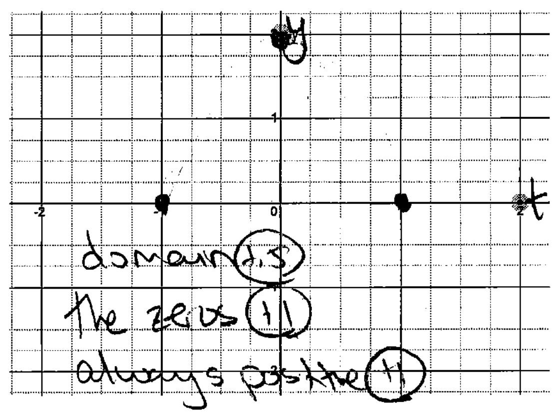
(+1) → $\approx (.15, 1), (-.15, 1)$ or $t = .25$ and $.25$

(+.5) @ end points

$\approx (-1.5, 0), (1.5, 0)$ or $t = -1$ and 1

(b) [6] (WrittenHW§10.1#32) Sketch the equations $x = f(t)$ and $y = g(t)$ on the pair of axis below.





start (+.5)

(c) [4] (WebHW10.2#3) Given the following information, find the (approximate) line tangent to the curve $x = f(t)$, $y = g(t)$ when $t = \frac{1}{2}$. Use whatever form of a line you like (eg. parametric, slope-intercept, standard, etc)

start (+.5)

$f\left(\frac{1}{2}\right) \approx 0 \qquad g\left(\frac{1}{2}\right) \approx .45 \qquad f'\left(\frac{1}{2}\right) \approx 1 \qquad g'\left(\frac{1}{2}\right) \approx -2.68$

any line eq. (+.5) { Looking for $y - y_0 = m(x - x_0)$

(+.5) { $m =$ slope of line tangent to graph @ $t = \frac{1}{2}$ $= \left.\frac{dy}{dx}\right|_{t=\frac{1}{2}}$

so thru $(x_0, y_0) = (f(\frac{1}{2}), g(\frac{1}{2})) = (0, .45)$ } (+.5)

(+1) { $= \frac{\left.\left(\frac{dy}{dt}\right)\right|_{t=\frac{1}{2}}}{\left.\frac{dx}{dt}\right|_{t=\frac{1}{2}}} = \frac{g'(\frac{1}{2})}{f'(\frac{1}{2})}$

$= \frac{-2.68}{1}$

So

$y - .45 = -2.68(x - 0)$ } (+1) plug in

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