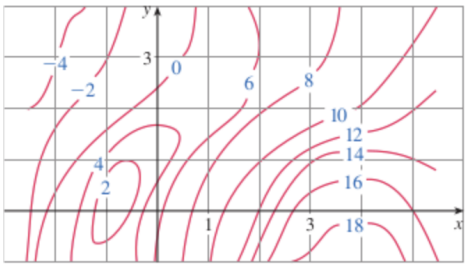
Exam 1 **TMath 126** Autumn 2023

1. [12] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

   (a) (Suggested §14.3#6) A contour map is given for a function $f$ below. This map implies $f_x(2,1) \approx -2$.




   (b) (dotActivity#1) If $\vec{w}$ and $\vec{v}$ are vectors in 3D, then $(\vec{w} \cdot \vec{j}) + \vec{v}$ returns a vector.

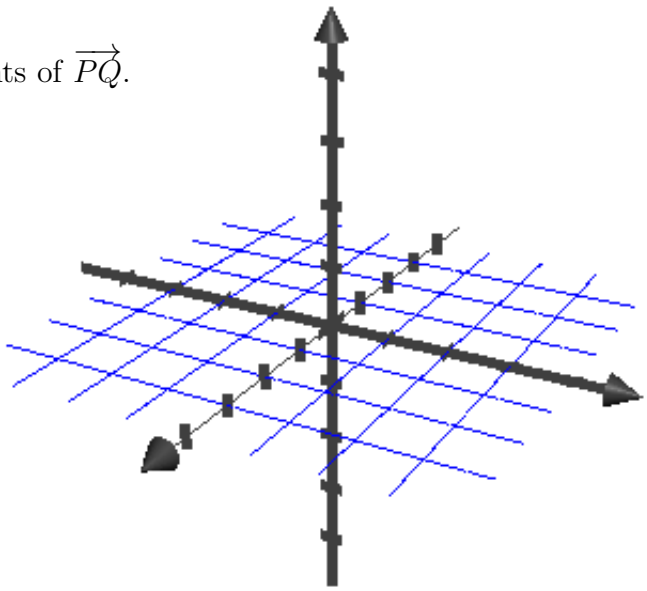
   (c) (WebHW14.2#2) The limit $\lim\limits_{(x,y)\to(\frac{3\pi}{2},\pi)} y\sin(x-y) = \pi$

   (d) (§13.2#26) If $\vec{r}(t) = \langle 2^t, \ln(t+1), t\rangle$, then the line tangent to $\vec{r}(0)$ is:

$$\langle 1,0,0\rangle + \langle 2^t \ln(2), \frac{1}{1+t}, 1\rangle$$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider the points $P(0,0,3)$ and $Q(-2,3,0)$



(a) [1] (PracticeExam1#2) Find the components of $\overrightarrow{PQ}$.

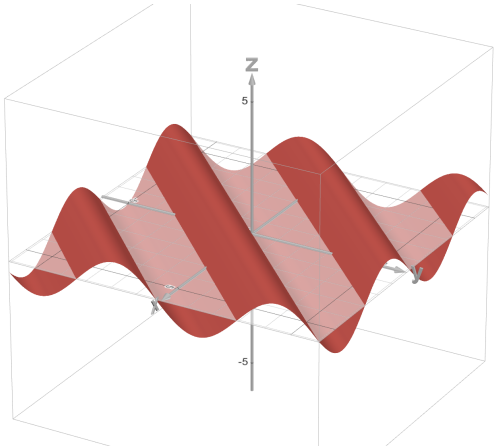
(b) [2] (DotActivity#2)
Find a vector parallel to $\overrightarrow{PQ}$.

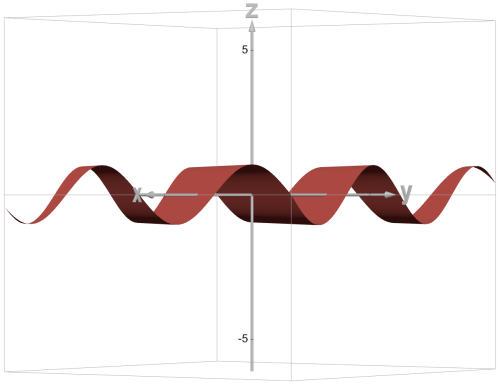
(c) [3] (Quiz2#1) Find the angle $\overrightarrow{PQ}$ makes with $\langle 0,1,3\rangle$.

(d) [3] (WebHW12.5 #4) Find an equation of a plane passing through $(2,1,0)$ and normal/orthogonal/perpendicular to $\overrightarrow{PQ}$

3. [3] (§14.1#64) Two perspectives of the graph of $f(x, y)$ are shown below. Identify which algebraic rule below corresponds with it. Provide justification!!!

- $f(x, y) = \sin(x) - \sin(y)$
- $f(x, y) = \sin(xy)$
- $f(x, y) = \sin(x - y)$

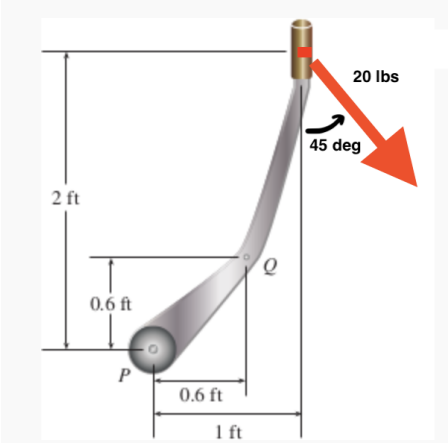



4. Consider the bicycle pedal shown on the right. A horizontal force of 20 lbs is applied to the handle as shown.

(a) [2] (3DActivity #1) Identify a 3D axis on the picture indicating the positive $x$, $y$, and $z$ axis.

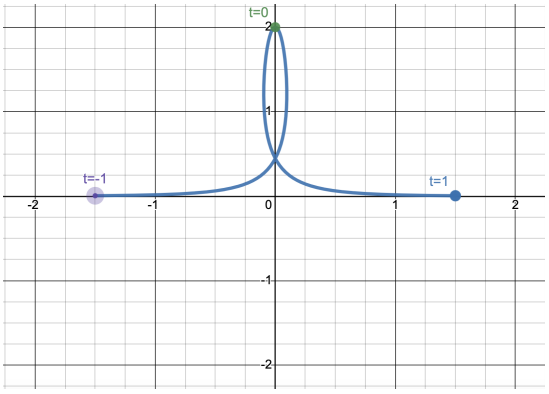
(b) [3] (WrittenHW12.4#40) Write the components of the force vector with respect to your 3D axis.



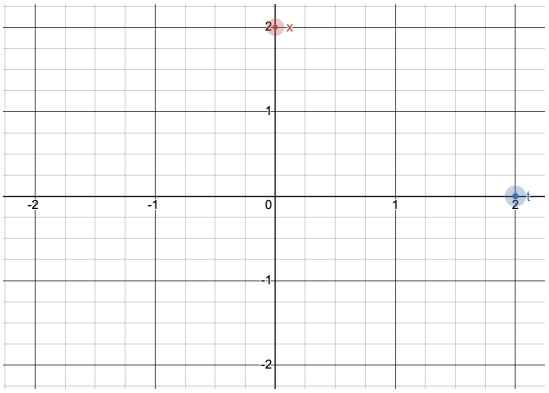

(c) [3] (Quiz2#2) Find the *vector* of the torque created about the pivot point $P$.

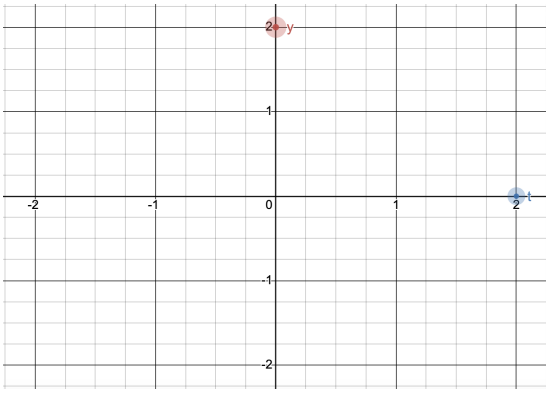
5. Consider the parametric curve $x = f(t)$, $y = g(t)$ where $-1 \leq t \leq 1$, graphed below for the following questions.

(a) [3] Looking at the graph, approximate where $\frac{dy}{dx}$ is not defined. (Report either a point on the graph or an approximate $t$ value.)




(b) [6] (WrittenHW§10.1#32) Sketch the equations $x = f(t)$ and $y = g(t)$ on the pair of axis below.




(c) [4] (WebHW10.2#3) Given the following information, find the (approximate) line tangent to the curve $x = f(t)$, $y = g(t)$ when $t = \frac{1}{2}$. Use whatever form of a line you like (eg. parametric, slope-intercept, standard, etc)

$f\left(\frac{1}{2}\right) \approx 0$ $\quad g\left(\frac{1}{2}\right) \approx .45$ $\quad f'\left(\frac{1}{2}\right) \approx 1$ $\quad g'\left(\frac{1}{2}\right) \approx -2.68$