Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. Each of the following is wrong. Explain why.

(a) Substitution yields:
$$\int_0^1 y(y^2+1)^5 dy = \int_0^1 \frac{1}{2}u^5 du$$

(b)
$$\int_{-1}^{1} \frac{1}{x^2} dx = \frac{-1}{x} \Big|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -2$$

- 2. Technical communication questions:
 - (a) How would you evaluate $\int \tan^a(x) \sec^b(x) dx$ if a is odd and b is even?

(b) Provide a second way of evaluating $\int \tan^a(x) \sec^b(x) dx$ if a is odd and b is even.

3. For each of the following outline the method(s) you would use to find the general antiderivative. Choose *THREE* to evaluate. No, you do not get extra credit for evaluating more than one.

$$\int x 4^x \, dx \qquad \qquad \int e^{\cos(t)} \sin(t) \cos(t) \, dt$$

$$\int 3\cot^3 x \, dx \qquad \qquad \int \frac{dy}{(y^2 + 36)^{\frac{3}{2}}}$$

$$\int \frac{3 + \sqrt{t} + t}{t} dt \qquad \qquad \int \frac{(\ln(x))^2}{x^2} dx$$

4. Let f(1) = 2, f(4) = 7, f'(1) = 5, f'(4) = 3, and assume f" is continuous.
(a) Evaluate \$\int_1^4 f'(x) dx\$.

(b) Evaluate
$$\int_1^4 f''(x) dx$$
.

(c) Evaluate
$$\int_{1}^{4} x f''(x) dx$$
.

5. The region under the curve $y = \cos^2(x)$ from $0 \le x \le \pi$ is rotated about the x-axis, find the volume of the resulting solid.

- 6. Consider the graphs depicting the median income for white Americans (top/red line) and Black Americans (bottom/blue line). The red line is described by the equation y = 566x + 40,738. The blue line is described by the equation y = 390x + 21,970.
 - (a) What is the meaning of the area between the red and blue lines from x = 0 to x = 100 (where x is years since 1960).



- (b) Estimate the area identified in part a. Clearly indicate your process.
- (c) Find the area between the curves from x = 0 to x = 50.

(d) Find the area between the curves from x = 50 to x = 100.

(e) Is income inequality between the two races growing or shrinking? Justify your answer.

- 7. Let $f(x) = \sin(x), g(x) = \sin^3(x).$
 - (a) Sketch a graph of f and g.
 - (b) Find the area trapped between the graph of f and g from x = 0 to $x = \pi$.

(c) Consider the object whose base is bounded by f, the x axis, x = 0 and $x = \pi$. The cross sections (perpendicular to the x axis) of the object are squares. Set up the integral to find the volume of this object.