

Show all your work.

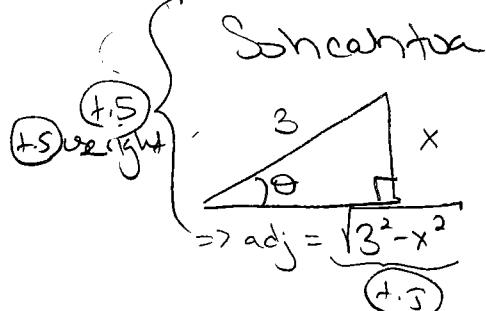
Reasonable supporting work must be shown to earn credit.

1. [4] (ActivityTrigSub #1) One problem required a substitution of $x = 3 \sin(\theta)$. Find the following in terms of x .

$$(a) \csc(\theta) = \frac{1}{\sin \theta} = \frac{1}{\frac{x}{3}} = \frac{3}{x}$$

$$(b) \cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{9-x^2}}{3}$$

$$\frac{x}{3} = \sin \theta$$



2. Each of the following is wrong. Explain why.

$$(a) [2] (\text{Quiz5}\#1) \int u \, dv = uv - v \, du$$

The integration by parts formula is $uv - \int v \, du$

In particular there should be an integral sign copied with the du

$$(b) [2] (\text{WebHW7-3}\#1) \int_1^2 \frac{4}{\sqrt{4+x^2}} dx = \int_1^2 \frac{8 \sec^2(\theta)}{\sqrt{4+(2 \tan(\theta))^2}} d\theta \text{ where } x = 2 \tan(\theta).$$

The bounds initially are with respect to x , when we convert to $d\theta$ we need to change the bounds to the corresponding θ 's.

$$2 = 2 \tan \theta \\ 1 = \tan \theta \Rightarrow \theta = \frac{\pi}{4}$$

$$1 = 2 \tan \theta \\ \frac{1}{2} = \tan \theta \Rightarrow \theta = \arctan(\frac{1}{2})$$

$$(c) [2] (\text{WrittenHW7-2}\#4) \int_0^{\frac{\pi}{4}} \sin^5(x) dx = \int_0^1 u^5 du \text{ where } u = \sin(x).$$

Start (+,5)

The substitution failed to replace the dx correctly.

Since $u = \sin(x)$, $du = \cos(x)dx \Rightarrow dx = \frac{1}{\cos(x)} du$

which was not taken into account here.

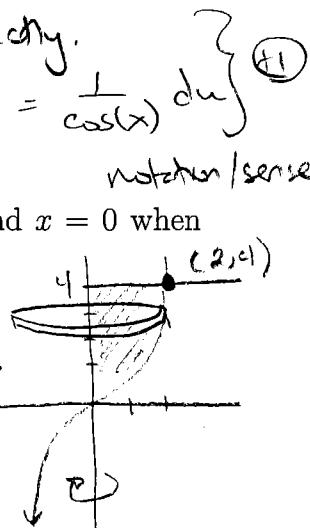
Notation/sense (+,5)

Notation/sense (+,5)

- (d) [2] (ActivityVolume) The region bounded by $y = \frac{1}{2}x^3$, $y = 4$, and $x = 0$ when revolved around the y -axis has volume equal to $\int_0^2 \pi(\sqrt[3]{2y})^2 dy$

Start (+,5)

(+) { The approx. cylinders have y vary from 0 to 4.
It looks like the limits (0 to 2) were taken with respect to x by mistake.



Notation/sense (+,5)

Types?

3. A particle is moving along a straight line with velocity $v(t) = \sin(\frac{t}{\pi}) \cos(\frac{t}{\pi})$ measured in meters per second.

- (a) [4] (WebHW7-2#9) Find the position function of the particle if we know at time 0, the position is 0.

Start (+5)
use integrals (+5)
notation (+5)

$$\text{Position} = \text{velocity} \cdot \text{time} = \int_0^T v(t) dt = \int_0^T \sin\left(\frac{t}{\pi}\right) \cos\left(\frac{t}{\pi}\right) dt$$

Note: there are MANY different ways to do this?

$$u = \sin\left(\frac{t}{\pi}\right) \quad du = \frac{1}{\pi} \cos\left(\frac{t}{\pi}\right) dt$$

$$= \int_0^{\sin(\frac{T}{\pi})} u \cdot \pi du = \pi \left[\frac{1}{2} u^2 \right]_0^{\sin(\frac{T}{\pi})}$$

$$= \frac{\pi}{2} \sin^2\left(\frac{T}{\pi}\right) + C \quad b/c @ T=0 \text{ position}=0 \Rightarrow C=0$$

- (b) [3] (WordProblem2#1) Find the instantaneous acceleration of the particle after one minute.

Recall acceleration = $\frac{d}{dt}$ (velocity) (+1)

$\begin{aligned} &= \frac{d}{dt} (\sqrt{t}) \\ &= \sin\left(\frac{t}{\pi}\right) \sin\left(\frac{t}{\pi}\right) \cdot \frac{1}{\pi} + \cos\left(\frac{t}{\pi}\right) \cdot \frac{1}{\pi} \cos\left(\frac{t}{\pi}\right) \end{aligned}$

4. Let g be the line graphed below on the right. Let f be a function that is continuous and twice differentiable to continuous functions. Assume that we also have the following values for f and f' .

x	$f(x)$	$f'(x)$
0	2	3
4	7	5

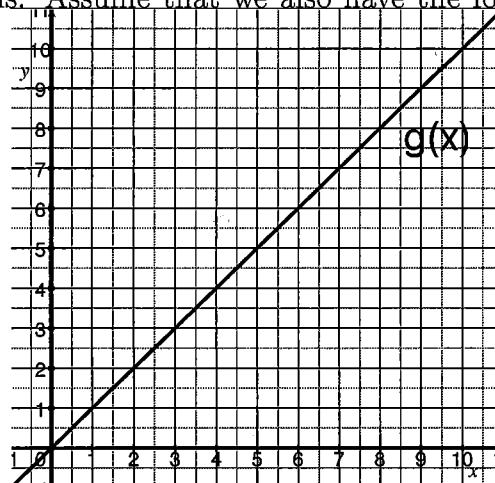
- (a) [1] Find $f(4)$.

$$7 \quad (+1)$$

- (b) [2] Find $g'(4)$.

sloped line tangent to g (+1)

$$= \frac{\Delta y}{\Delta x} = 1 \quad (+1)$$



- (c) [3] (practiceExam2 #4) Evaluate $\int_0^4 f'(x) dx = [f(x)]_0^4 = f(4) - f(0) = 7 - 2 = 5$

start (+5)

by
FTC
(+5)
an antiderivative
(+5)

- (d) [4] (practiceExam2 #4) Evaluate $\int_0^4 g(x)f''(x) dx = \int_0^4 x f''(x) dx$

note $g(x)=x$ so we can simplify to:

$\begin{cases} \text{Integration by Parts} \\ u = x \\ v = f'(x) \\ dv = f''(x) dx \end{cases}$

$$\begin{aligned} &= x[f'(x)]_0^4 - \int_0^4 f'(x) dx \quad \{ (+5) \\ &= [4f'(4) - 0f'(0)] - [f(x)]_0^4 \\ &= [4f'(4) - 0f'(0)] - [f(x)]_0^4 \\ &= [4f'(4) - 0f'(0)] - [f(x)]_0^4 \\ &= 4f'(4) - [f(x)]_0^4 \\ &= 4f'(4) - [f(4) - f(0)] \\ &= 4f'(4) - [7 - 2] \\ &= 4f'(4) - 5 \\ &= 4(5) - 5 \\ &= 20 - 5 = 15 \end{aligned}$$

5. For each of the following, identify the technique you would use to find the indefinite integral. For example, if you think substitution would work, write "substitution" and identify what u would be. If you think integration by parts, write "integration by parts" and identify what u and dv would be.

(a) [2]

Start $\frac{d}{dt} \sin(t)$
a method $\frac{d}{dt}$
works $\frac{d}{dt}$

$$\int \cos^5(t) dt$$

Substitution $u = \sin(t)$
use pythagorean to transfer $\cos^4(t)$ into $(1 - \sin^2 t)^2$ then start.

$$\begin{aligned} \sin^2 t + \cos^2 t &= 1 \\ \Rightarrow \cos^2(t) &= 1 - \sin^2 t \end{aligned}$$

(b) [2]

Start $\frac{d}{dt} \sqrt{y-4}$
a method $\frac{d}{dt}$
works $\frac{d}{dt}$

$$\int y \sqrt{y-4} dy = \int (u+4) u^{\frac{1}{2}} du$$

Substitution $u = y-4$ and $y = u+4$ Distribution will give us something easy to integrate.
 $du = dy$

(c) [2]

Start $\frac{d}{dx} \sin(x)$
a method $\frac{d}{dx}$
works $\frac{d}{dx}$

Integration by parts
 $u = x$ $v = -\cos(x)$
 $du = dx$ $dv = \sin(x) dx$

6. [4] Evaluate one of the indefinite integrals above.

$$\begin{aligned} (a) \int \cos^5(t) dt &= \int \cos^4(t) \cos(t) dt \\ &= \int (1 - \sin^2(t))^2 \cos(t) dt \quad u = \sin(t) \\ &= \int (1 - u^2)^2 du \quad du = \cos(t) dt \\ &= \int (1 - u^2)(1 - u^2) du = \int 1 - 2u^2 + u^4 du \\ &= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C \\ &= \sin(t) - \frac{2}{3}\sin^3(t) + \frac{1}{5}\sin^5(t) + C \end{aligned}$$

Check: $[\sin(t) - \frac{2}{3}\sin^3(t) + \frac{1}{5}\sin^5(t) + C]'$

$$\begin{aligned} &= \cos(t) - \frac{2}{3} \cdot 3\sin^2(t) \cdot \cos(t) + \frac{1}{5} \cdot 5\sin^4(t) \cos(t) \\ &= \cos(t) [1 - 2\sin^2(t) + \sin^4(t)] \\ &= \cos(t) (1 - \sin^2(t))(1 - \sin^2(t)) \\ &= \cos(t) \cos^2(t) \cos^2(t) \\ &= \cos^5(t) \checkmark \end{aligned}$$

$$\begin{aligned} (b) \int y \sqrt{y-4} dy &\text{ let } u = y-4 \Rightarrow y = u+4 \\ &= \int (u+4) u^{\frac{1}{2}} du \\ &= \int u^{\frac{3}{2}} + 4u^{\frac{1}{2}} du = \frac{2}{5}u^{\frac{5}{2}} + \frac{8}{3}u^{\frac{3}{2}} + C \\ &= \frac{2}{5}(y-4)^{\frac{5}{2}} + \frac{8}{3}(y-4)^{\frac{3}{2}} + C \end{aligned}$$

Check: $[\frac{2}{5}(y-4)^{\frac{5}{2}} + \frac{8}{3}(y-4)^{\frac{3}{2}} + C]'$
 $\frac{2}{5} \cdot 2(y-4)^{\frac{3}{2}} + \frac{8}{3} \cdot \frac{3}{2}(y-4)^{\frac{1}{2}} + 0 = (y-4)^{\frac{3}{2}} + 4(y-4)^{\frac{1}{2}}$
 $= (y-4)^{\frac{1}{2}} [(y-4)^{\frac{1}{2}} + 4] = (y-4)^{\frac{1}{2}} (y-4+4) \checkmark$

$$(c) \int x \sin(x) dx \quad \begin{array}{l} \text{Integration by parts} \\ u = x \quad v = \int \cos(x) \\ du = dx \quad dv = \sin(x) dx \end{array}$$

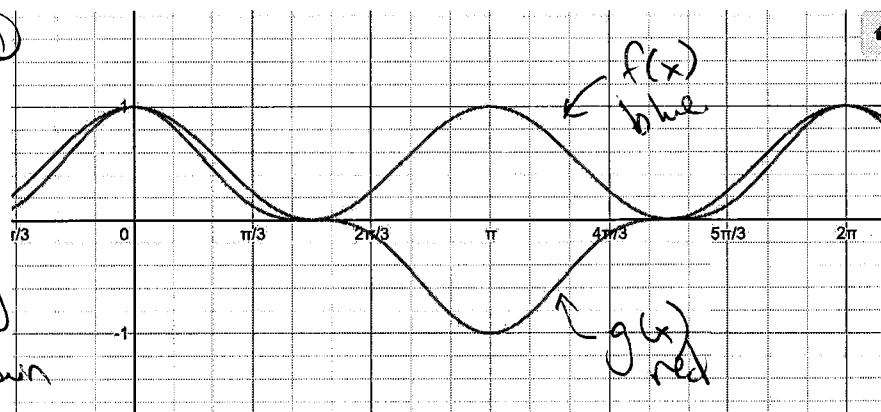
$$x \cos(x) + \int \cos(x) dx = -x \cos(x) + \sin(x) + C$$

Check: $[-x \cos(x) + \sin(x) + C]'$
 $= -x \sin(x) + (-1) \cos(x) + \sin(x) + 0$
 $= x \sin(x) \checkmark$

7. (WebHW7-2) Consider the curves $f(x) = \cos^2(x)$, $g(x) = \cos^3(x)$ graphed below.

(a) [2] Identify (1) which graph is f and which is g .

Justify your answer.



$f(x)$ is a quantity squared so the y-value will remain positive/above the x-axis.

(b) [2] Set up the definite integral that would evaluate the area bounded by f , g , and the vertical lines $x = 0$ and $x = 2\pi$. Do not compute this!!

$$\int_0^{2\pi} f(x) - g(x) dx = \int_0^{2\pi} \cos^2(x) - \cos^3(x) dx$$

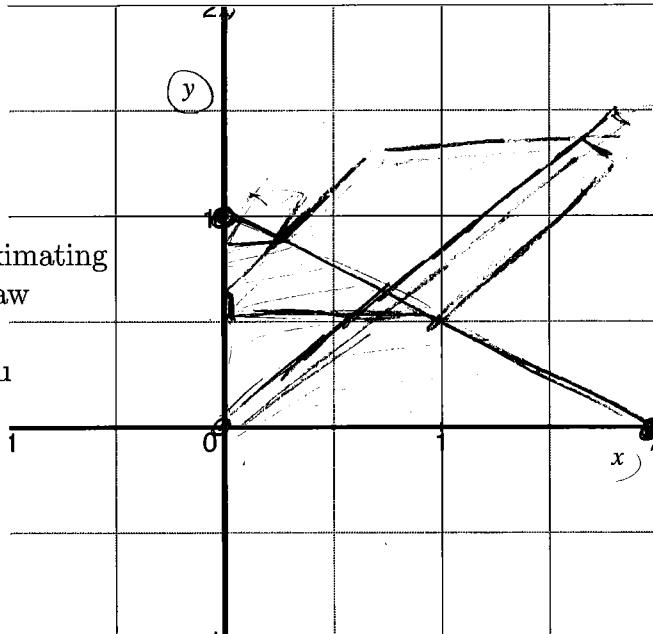
bands (1)

bigger-smaller (1)

8. (SuggestedWrittenHW6-2#69) Define an object, S , with a triangular base in the xy plane with vertices $(0, 0)$, $(2, 0)$, and $(0, 1)$. The cross-sections of S perpendicular to the y -axis form squares.

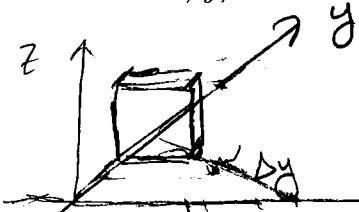
(a) [2] Sketch the base of the object S on the xy plane to the right.

Dotted each point (1.5)
start (1.5)



X,y,z axis (1.5)

Square cylinder (1.5)
sketched correctly (1.5)



(c) [3] Set up the definite integral that would find the volume of S . Do not compute this!!

take & take limit? use the line equation

Sum of square cylinders

Sum of $(\text{side length})^2 \Delta y$

Sum of $(x\text{-coord})^2 \Delta y$

Sum of $(2y-2)^2 \Delta y$

$$y = \left(\frac{-1}{2}\right)x + 1$$

$$y - 1 = -\frac{1}{2}x$$

$$-2(y - 1) = x$$

$$-2y + 2 = x$$