Key

Exam 1 — TMath 125 — Winter 2022

Show all your work.
Reasonable supporting work must be shown to earn credit.

1. [3] (SummationActivity #1) Expand $\sum_{i=2}^{6} \left(\frac{(-1)^i}{i-3} \right)$.

   (You do *not* need to compute or simplify this!)

   terms (+2) start (+.5) end (+.5)

   $\underset{i=2}{\frac{(-1)^2}{2-3}} + \underset{i=3}{\frac{(-1)^3}{3-3}} + \underset{i=4}{\frac{(-1)^4}{4-3}} + \underset{i=5}{\frac{(-1)^5}{5-3}} + \underset{i=6}{\frac{(-1)^6}{6-3}}$

   ↳ would be a problem to evaluate?

2. Find the following.

   (a) [2] (Week2Monday) $\int \sin(t)\,dt$

   family of functions answer (+.5)

   $-\cos(t) + C$ (+.5) (+.5) (+.5)

   Check $\frac{d}{dt}(-\cos t) = +\sin(t)$

   (b) [4] (WebHW5-4&5-3 #7) $\int_1^4 \frac{3+\sqrt{x}+x}{x}\,dx = \int_1^4 \frac{3}{x} + \frac{\sqrt{x}}{x} + \frac{x}{x}\,dx$ alg (+.5)

   number answer (+.5)

   $= \int_1^4 3x^{-1} + x^{-1/2} + 1\,dx = 3\ln(x) + 2x^{1/2} + x \Big]_1^4$ simplify (+.5) (+.5) (+.5) (+.5)

   $= (3\ln 4 + 2\cdot\sqrt{4} + 4) - (3\ln 1 + 2\sqrt{1} + 1) = 3\ln 4 + 8 - 3$ plug in values (+1)

   $= 3\ln 4 + 5 \approx 9.16$

   (c) [4] (WrittenHW5-5 #90) $\int \frac{2e^{0.4x}}{(1+5e^{0.4x})^2}\,dx$

   family of functions answer (+.5)

   sub (+.5)

   (+.5) $u = 1 + 5e^{0.4x}$

   (+.5) $du = 0 + 5e^{.4x} \cdot .4\,dx$

   $du = 2e^{.4x}\,dx$

   $\int \frac{du}{u^2} = \int u^{-2}\,du$

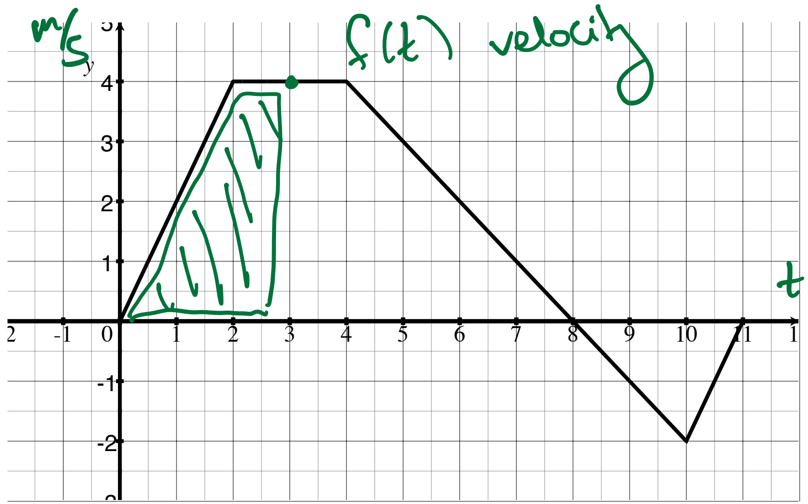
   $= -u^{-1} + C = \frac{-1}{1+5e^{0.4x}} + C$ (+1) (+.5)

   notation (+.5)

   Check: $\frac{d}{du}(u^{-1}) = -u^{-2}$ ✓

3. Let $f(t)$ be the piece-wise defined function graphed below that is comprised of straight lines. The graph of f reports the velocity (m/s) of an electric vehicle moving on a straight track after t seconds. At $t = 0$, the vehicle is at the charging station.

Let $g(x) = \int_0^x f(t)\,dt$



(a) [1] (Quiz1#1a) Estimate $f(3)$.

graph reading (+.5)

4 m/s (+.5)

(b) [1] (WebHW5-3#7) Estimate $f'(3)$.

(+.5) slope of line tang. to f @ t=4

0 (+.5)

(c) [2] (WrittenHW5-3#4) Find $g(3)$, exactly.

$g(3) = \int_0^3 f(t)\,dt$ (+.5) = area shaded (+.5) $= \frac{1}{2}(2)(4) + 1 \cdot 4$

$= 4 + 4 = 8$ (+1)

(d) [2] (WebHW5-4&5-3#9) Interpret $g(3)$ in terms of distance or velocity of the electric vehicle.

sense (+.5)

The electric vehicle traveled 8 meters in the first 3 seconds. (+1) (+.5)

(e) [2] (WrittenHW5-3#4) Estimate $g'(3)$.

(+.5)

$g'(3) = \frac{d}{dx}\int_0^x f(t)\,dt\Big|_{x=3} = f(t)\Big|_{x=3} = f(3)$

(+.5) (FTC I) (+.5)

$= 4$ m/s (+.5)

(f) [3] (WrittenHW5-4 #68, WrittenHW5-3#12) At what time is the vehicle farthest from the charging station? Justify your answer.

start (+.5)
sense (+.5)

@ 8 seconds (+1)

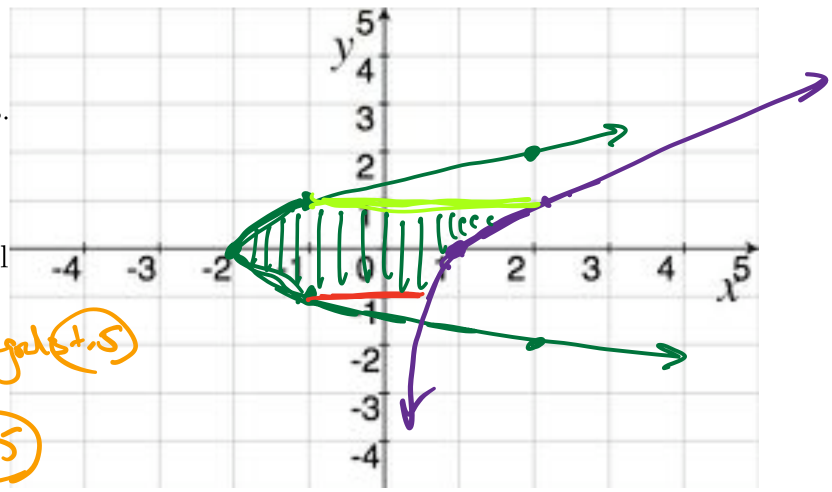
before 8 seconds the velocity is positive but after 8 seconds the vehicle starts traveling backwards / ie getting closer to the station (+1)

4. (SuggestedHW6-1#3) Consider the area trapped by $f(y) = y^2 - 2$, $g(y) = e^y$, $y = -1$, and $y = 1$.

+1 +1 +.5

+.5

(a) [3] Sketch and shade the region bounded by the curves.



(b) [4] Set up the definite integral (but do *not* compute!)that will find the area of the shaded region above.

integrals +.5

easier if we do with respect to y +.5

+1 +.5 +.5

$$\int_{-1}^{1} (e^y) - (y^2 - 2)\,dy$$

$e^y = g(y)$ first +.5

$y^2 - 2$ is in there +.5

5. Let $g(t)$ be a continuous function such that $\int_{-3}^{1} g(t)\,dt = 3$ and $\int_{1}^{4} g(t)\,dt = -1$. Find the following:

(a) [2] (DefiniteIntegralActivity#3) $\int_{-3}^{4} 5g(t)\,dt$

try int prop +.5

+.5 +.5

$$\int_{-3}^{4} 5g(t)\,dt = 5\int_{-3}^{4} g(t)\,dt = 5\left(\int_{-3}^{1} g(t)\,dt + \int_{1}^{4} g(t)\,dt\right)$$

$$= 5(3 + -1) = 5 \cdot 2 = 10$$

+.5

(b) [3] (Quiz1#2) $\int_{-3}^{1} g(t) + 1\,dt$

start +.5
notation +.5

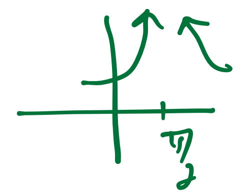
+.5

$$\int_{-3}^{1} g(t) + 1\,dt = \int_{-3}^{1} g(t)\,dt + \int_{-3}^{1} 1\,dt$$

+.5

$$= 3 + \underbrace{1\,(1 - {-3})}_{+1} = 3 + 4 = 7$$

6. Each of the following is wrong. Explain why.



(a) [2] (Written5-3#66) $\int_0^{\pi} \sec^2(x)\,dx = \tan(x)|_0^{\pi} = 0$

start (+.5)

We cannot use the Fund. Thm. of Calc II on functions that are not continuous on the domain - like $\sec^2 x$

+1 +.5

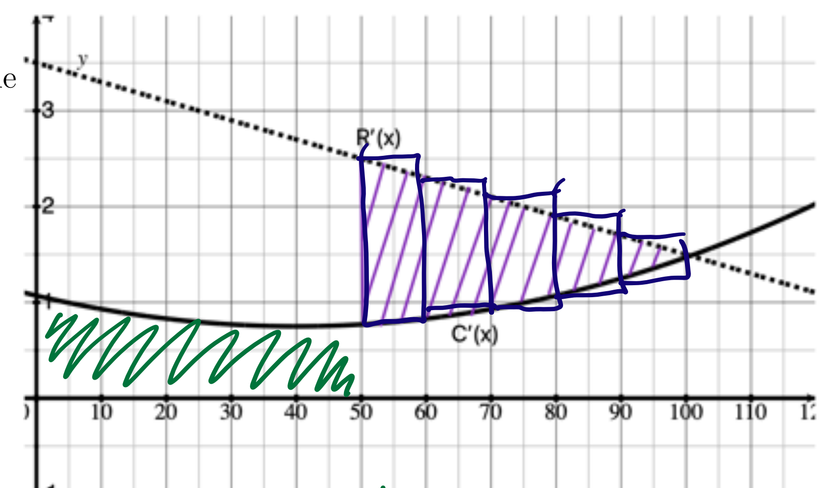
(b) [2] $\int_1^2 \frac{4}{x^3}\,dx = \int_1^2 4x^{-3}\,dx = 4(-3)x^{-4}|_1^2 = -12 \cdot 2^{-4} - (-12 \cdot (1)^{-4}) = -192 + 12 = 180$

sense (+.5)

The Fund. Thm of Calc II needs an Antiderivative. We found the derivative of $4x^{-3}$ instead.

+1 +.5

7. The graph below shows the marginal revenue function $R'(x)$ and the marginal cost function $C'(x)$ for a manufacturer. Assume that R and C are measured in thousands of dollars.



(a) [2] (Quiz1#1) Shade the region described by $\int_0^{50} C'(x)\,dx$.

left/right +1

upper/x-axis +1

green

(b) [3] (PracticeExam#11) What is the meaning of the area of the shaded region?

The profit of making the 2nd set of 50 items.

sense (+.5) +1 +.5 +1

involves revenue & cost +1

(c) [3] (WrittenHW5-1 #14) Approximate the area of the shaded region. Make sure it is clear what your approximation technique is!

I'll use rectangles - left handed ones +1

$\approx 1.7 \cdot 10 + 1.3 \cdot 10 + 1.1 \cdot 10 + .9 \cdot 10 + .5 \cdot 10 = 55$ thousand +.5

graph reading +1 Δx's +.5

8. [2] What concept did you study but not see on the exam?

+2 Forced left/right hand approximations with rectangles

↳ note anything in the math curriculum should work here