Key

TMATH 125 Quiz 3

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [3] (WebHW8#3) Find $\int_0^2 s \cdot 3^s \, ds$.

try IP (+.5)
do IP (+.5)

$u = s \qquad v = \frac{1}{\ln 3} 3^s$ (+.5)

$du = ds \qquad dv = 3^s ds$

$$\int_0^2 s\, 3^s ds = uv\Big]_0^2 - \int_0^2 v\, du$$

$$= s \frac{1}{\ln 3} 3^s \Big]_0^2 - \int_0^2 \frac{1}{\ln 3} \cdot 3^s ds$$

plug in values (+1)

$$= \left[2 \frac{1}{\ln 3} \cdot 3^2 - 0 \frac{1}{\ln 3} \cdot 3^0\right] - \frac{1}{\ln 3} \int_0^2 3^s ds$$

$$= \left[\frac{18}{\ln 3} - \frac{0}{\ln 3}\right] - \underbrace{\frac{1}{\ln 3} \cdot 3^s \cdot \frac{1}{\ln 3}\Big]_0^2}_{(+.5)}$$

$$\left[\frac{18}{\ln 3} - \frac{0}{\ln 3}\right] - \left[\frac{1}{\ln 3} \cdot 3^2 \cdot \frac{1}{\ln 3} - \frac{1}{\ln 3} \cdot 3^0 \frac{1}{\ln 3}\right]$$

$$\frac{18}{\ln 3} - \left[\frac{9}{(\ln 3)^2} - \frac{1}{(\ln 3)^2}\right]$$

$$\frac{18}{\ln 3} - \frac{8}{(\ln 3)^2}$$

$\approx$

2. [2] (WebHW8 #7) Consider the region bounded by the curves $y = 6x^2 \ln x$ and $y = 24 \ln x$. Set up the integral (but do *not* integrate) that is used to find the this area.

(+.5) note the 2 curves intersect when $6x^2 \ln x = 24 \ln x$

$\Rightarrow \quad 6x^2 \ln x - 24 \ln x = 0$

$\Rightarrow \quad \ln x\,(6x^2 - 24) = 0$

$\Rightarrow \quad \ln x = 0 \quad$ or $\quad 6x^2 - 24 = 0$

(+.5) $\Rightarrow \quad x = 1 \qquad x^2 = 4$

$x = +2, \cancel{-2}$ (domain of ln)

notice $6(1.5)^2 \ln(1.5) < 24 \ln(1.5)$

So $\int_1^2$ top curve − bottom curve dx (+.5)

$$\int_1^2 24 \ln x - 6x^2 \ln x \, dx$$ (+.5)

3. [3] (Trig Wks #3) Find $\int \tan^6(x)\sec^4(x)\,dx$. $= \int \tan^6(x)\sec^2(x)\sec^2(x)\,dx$

let $u = \tan(x)$ trig sub (+.5)

$du = \sec^2(x)dx$ got sub (+.5)

Recall $\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$ (+.5)

$\Rightarrow \tan^2 x + 1 = \sec^2 x$

$\int \tan^6(x)\sec^2(x)\sec^2(x)\,dx$

use pyth (+.5)

$\int u^6(\tan^2 x + 1)\,du$

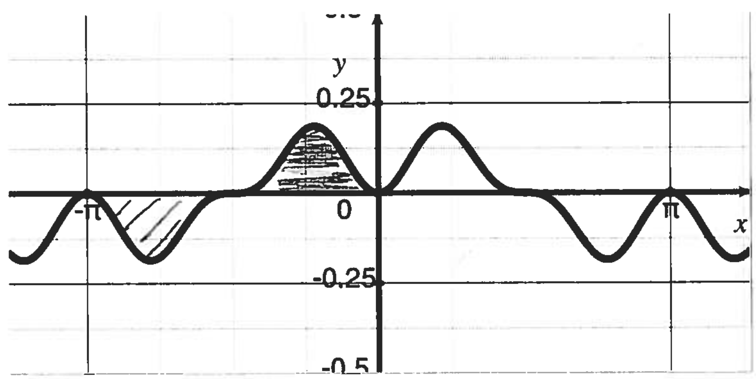
$\int u^6(u^2+1)\,du$

$\int u^8 + u^6\,du$

$\frac{1}{9}u^9 + \frac{1}{7}u^7 + c = \frac{1}{9}\tan^9(x) + \frac{1}{7}\tan^7(x) + c$

(+.5) (+.5)

4. Consider $f(x) = \sin^2(x)\cos^3(x)$ shown on the right.



(a) [1] (§7.2 #55) Set up the integral (but do *not* to find the average value of the function on the interval $[-\pi, \pi]$.

$$\frac{1}{\pi - -\pi}\int_{-\pi}^{\pi} \sin^2(x)\cos^3(x)\,dx$$

(b) [1] Use the graph and the definition of average value to find the average value of f on the interval $[-\pi, \pi]$.

zero b/c of symmetry (areas = areas)