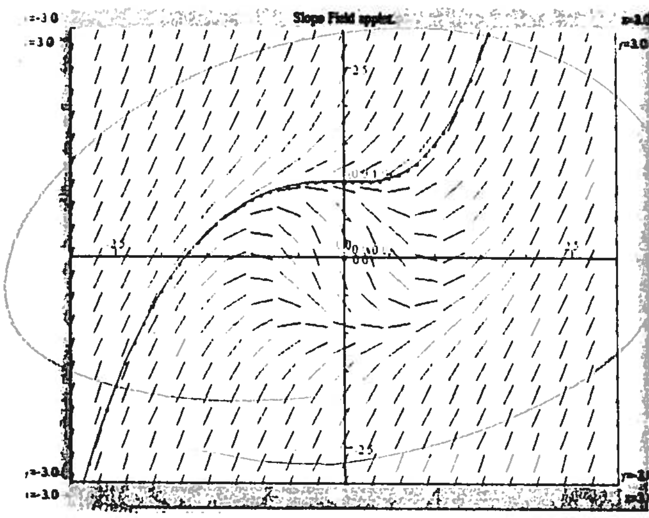
Key

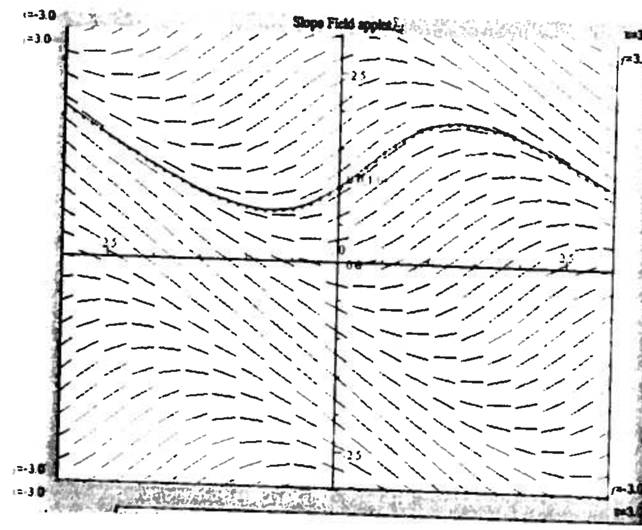
TMATH 125 Quiz 4

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [2] (§9.1 #12) Identify which function could be a solution to the differential equation $\frac{dy}{dx} = \log_2(x^2+y^2)$. Briefly *justify* your choice. (+1)



(+1)



Note $\left.\frac{dy}{dx}\right|_{y=1,\ x=0} = \log_2(0^2+1^2) = \log_2(1) = 0$ So the slope at $(0,1)$ is horizontal.

The graph on the right has a positive slope at $(0,1)$

2. [4] (sep dif eq wks #2) Find the solution of the differential equation that satisfies the given initial condition:

$$\frac{dy}{dt} = .00202y(3000-y) \qquad y(0) = 2$$

algebra (+.5)

separate (+.5)

$$\frac{1}{y(3000-y)}\frac{dy}{dt} = .00202\ dt$$

$$\int \frac{1}{y(3000-y)}\,dy = \int .00202\,dt$$

need partial fractions..

(.5)

$$\frac{A}{y} + \frac{B}{3000-y} = \frac{1}{y(3000-y)} \Rightarrow A = \tfrac{1}{3000} \text{ and } B = \tfrac{1}{3000}$$

$\Rightarrow A3000 - Ay + By = 1$

$\Rightarrow -A+B=0$ and $3000A = 1$

$$\int \frac{\frac{1}{3000}}{y} + \frac{\frac{1}{3000}}{3000-y}\,dy = \int .00202\,dt$$

(+.5) (+.5) (+.5) (+.5)

$$\frac{1}{3000}\int \frac{1}{y}\,dy + \frac{1}{3000}\int \frac{1}{3000-y}\,dy = .00202t + C$$

$$\frac{1}{3000}\left[\ln|y| - \ln|3000-y|\right] = .00202t + C$$

since $y(0)=2$ (+.5)

$$\frac{1}{3000}\left[\ln(2) - \ln(3000-2)\right] = .00202\cdot 0 + C$$

$$\Rightarrow C = \frac{1}{3000}\left[\ln 2 - \ln 2998\right]$$

$$\text{So } \frac{1}{3000}\left[\ln\left|\frac{y}{3000-y}\right|\right] = .00202t + \frac{1}{3000}\ln\left|\frac{2}{2998}\right|$$

3. [4] (WebHW13 #7) A vat with 200 gallons of beer contains 4% alcohol (by volume). Beer with 6% alcohol is pumped into the vat at a rate of 2 gal/min and the mixture is pumped out at the same rate. Let $A(t)$ be the amount of alcohol in the vat at time t and set up a differential equation modeling the described situation. Do *not* solve the differential equation, but do justify the differential equation you set up.

$A(t)$ is the amount of alcohol in the vat at time t.

(+.5) $\frac{dA}{dt}$ = rate that alcohol is changing with respect to time

(+1) = rate of alcohol coming in − rate of alcohol leaving

$$= \underbrace{.06 \frac{\text{gal alc}}{\text{gal } \cancel{\text{beer}}} \cdot 2 \frac{\text{gal } \cancel{\text{beer}}}{\text{min}}}_{(+1)} - \underbrace{2 \frac{\text{gal } \cancel{\text{beer}}}{\text{min}} \cdot \frac{A(t) \text{ gal alc}}{200 \text{ gal } \cancel{\text{beer}}}}_{(+1)}$$

$$= .12 \frac{\text{gal alc}}{\text{min}} - \frac{A(t)}{100} \frac{\text{gal alc}}{\text{min}}$$

So $$\frac{dA}{dt} = .12 - \frac{A(t)}{100} = \frac{12}{100} - \frac{A(t)}{100}$$ alg/got it (+.5)

$$\frac{dA}{dt} = \frac{12 - A}{100}$$

just for fun...

$$\frac{\frac{dA}{dt}}{12-A} = \frac{\frac{12-A}{100}}{12-A}$$

$$\cancel{dt}\ \frac{1}{12-A} \frac{dA}{\cancel{dt}} = \frac{1}{100}\, dt$$

$$\int \frac{1}{12-A}\, dA = \int \frac{1}{100}\, dt$$

$u = 12-A$

$du = -1 dA$

$$-\ln|12-A| = \frac{1}{100}t + c$$

which we could totally solve for A

$$\ln|12-A| = -\tfrac{1}{100}t - c$$

$$12 - A = e^{-\frac{t}{100} - c}$$

$$12 - e^{-t/100 - c} = A(t) \ldots$$