FINAL

Math 125

PRACTICE

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let a and b be constants with $a \leq b$ and $f(x)$ and $g(x)$ be continuous functions on $[a, b]$.

   T F We can differentiate any rudimentary collection of functions with calculus 1 methods.

   T F We can integrate any rudimentary collection of functions with calculus 2 methods.

   T F $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

   T F $\displaystyle\int_a^b f(x)g(x)\,dx = \int_a^b f(x)\,dx * g(x) + f(x) * \int_a^b g(x)\,dx$

   T F If f is continuous, then $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \lim_{t\to\infty}\int_{-t}^{t} f(x)\,dx.$

   T F If $\int_a^\infty f(x)\,dx$ and $\int_a^\infty g(x)\,dx$ are both convergent, then $\int_a^\infty f(x) + g(x)\,dx$ is convergent.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. *Carefully* write down the first Fundamental Theorem of Calculus.

3. Describe Simpson's Rule for approximating areas. (I don't want a formula here, but rather an explanation of where the formula comes from.)

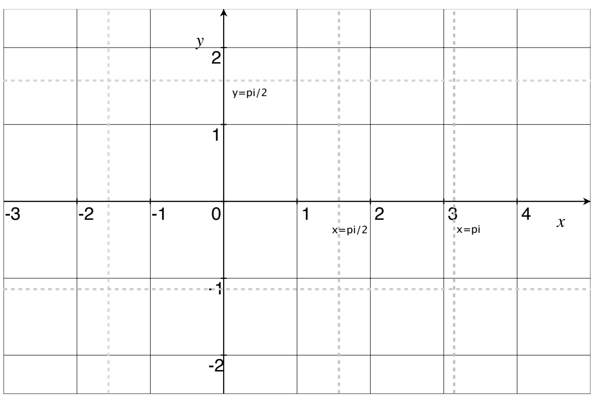
4. Find the following:

$$\frac{d}{dx}\int_x^3 \frac{3^u\pi - e}{\sqrt{u^3+7}}\,du \qquad\qquad \frac{d}{dx}\int_0^{x^2+3x} e^{t^2}\,dt$$

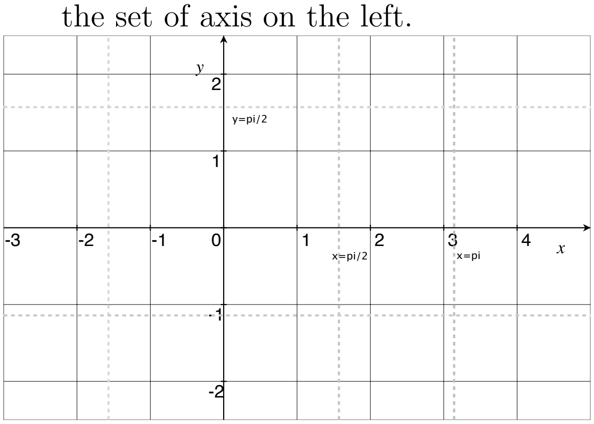
5. Let v be the function that records the velocity of a particle which is well approximated by the following formula.

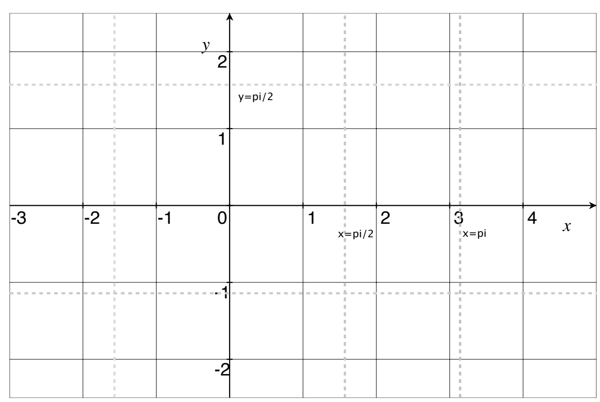
$$v(t) = \begin{cases} -2 & t \leq -1 \\ 2t & \text{if } -1 \leq x \leq 0 \\ \sin t & \text{if } 0 < t \end{cases}$$

(a) Carefully graph $v(t)$ on the set of axis.



(b) Give a rough sketch of the function recording the acceleration of the particle on the set of axis on the left.





(c) Give a rough sketch of the graph $\int_0^x v(t)\,dt$ on the set of axis on the right.

(d) Describe the physical meaning of $\int_0^x v(t)\,dt$.

6. For each of the following outline the method(s) you would use to find the general antiderivative. *For extra credit*, find the general antiderivative (each one will earn 1%).

$$\int_0^{\frac{\pi}{4}} \sec^4 x \tan^4 x \, dx$$

$$\int x \cos^2 x \, dx$$

$$\int_1^{\infty} \frac{1}{x^2} \, dx$$

$$\int \frac{1}{x^2\sqrt{x^2+4}} \, dx$$

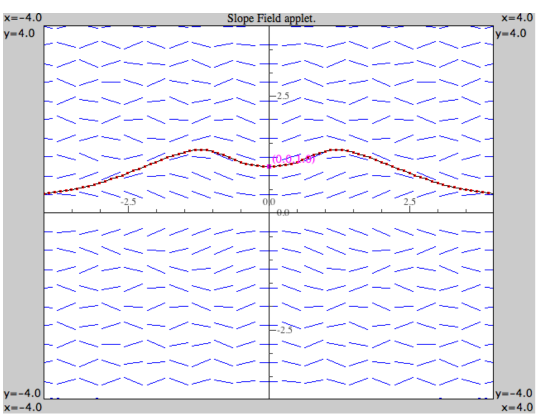
$$\int_0^{3} \frac{1}{x-1} \, dx$$

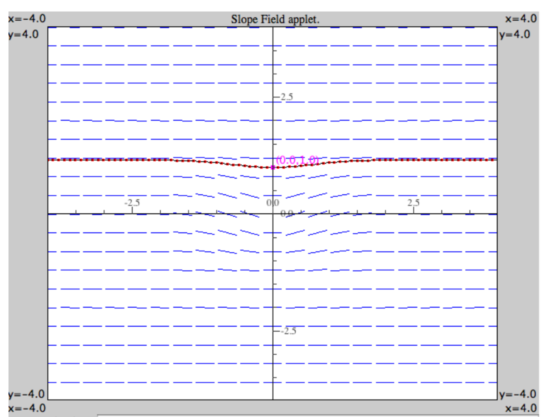
$$\int \frac{17x-1}{2x^2+3x-2} \, dx$$

7. Let $g(x) = \dfrac{12x}{x^2 + x - 2}$. Find the average value of g on the interval $[2, 5]$.

8. Match the differential equations with the solutions graphs.
Briefly *justify* your choice.

(a)$y' = xe^{-x^2-y^2}$ (b) $y' = \sin(xy)\cos(xy)$





9. Write the following in sigma notation:

$-\dfrac{1}{3} + \dfrac{3}{7} - \dfrac{1}{2} + \dfrac{5}{9} - \dfrac{3}{5} + \dfrac{7}{11}$ $1 + 2 + 4 + 8 + 16 + 32$

10. Let $f(x) = \sin(x)$. Find the area of the region bounded by f, $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x-axis.

11. Consider the region trapped between $f(x) = \frac{1}{x}$, the x-axis, and from $x = 0$ to $x = 1$.

    (a) If this region was revolved about the y-axis, what would the resulting volume be?

    (b) What would its volume be if it was revolved about the x-axis?

12. A tank has the shape of an inverted circular cone with height 10m and base 4 m. It is filled with water to a height of 8m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000kg/m^3.)

13. Dr. Card is found dead in his office at 5:00pm one evening. The temperature of his body was 80.0°F. One hour later, at 6:00pm, the body has cooled to 75.0°F. The room is kept at a constant temperature of 70°F. Assume Dr. Card had a normal temperature of 98.6°F at the time of death.

Let $f(t)$ be the temperature of the body after t hours.

(a) By Newton's law of cooling, the rate a body cools is proportional to the difference in temperature between the body and the ambient temperature. Write down the differential equation reflecting this particular situation.

(b) Solve for $f(t)$ as a function of t. .

(c) When did the murder take place?