Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. [] TRUE/FALSE: Circle $T$ in each of the following cases if the statement is always true. Otherwise, circle F.
$\mathrm{T} \quad \mathrm{F}$ If $f(1)=2, f(4)=7, f^{\prime}(1)=5, f^{\prime}(4)=3$, and $f^{\prime \prime}$ is continuous, we cannot evaluate $\int_{1}^{4} x f^{\prime \prime}(x) d x$.

T F Substitution yeilds: $\int_{0}^{1} y\left(y^{2}+1\right)^{5} d y=\int_{0}^{1} \frac{1}{2} u^{5} d u$
$\mathrm{T} \quad \mathrm{F} \quad \int_{-1}^{1} \frac{1}{x^{2}} d x=\left.\frac{-1}{x}\right|_{-1} ^{1}=\frac{-1}{1}-\frac{-1}{-1}=-2$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.
2. () Dr. Card and Dr. Eaton decide to have a short race. The following is a graph of their respective velocities at time $t$ measured in seconds.


(a) [2] Estimate the total distance each one runs during the race.
(b) [2] If the race is 20 ft , who wins the race? Explain how you know.
3. For each of the following outline the method(s) you would use to find the general antiderivative. For extra credit, find the general antiderivative (each one will earn $1 \%$ ).

$$
\int x 4^{x} d x \quad \int e^{\cos (t)} \sin (2 t) d t
$$

$$
\int 3 \cot ^{3} x d x
$$

$$
\int \pi\left(\frac{36}{y^{2}+36}\right)^{2} d y
$$

$$
\int \frac{t^{4}}{t-1} d t
$$

$$
\int \frac{x^{3}-4 x-10}{x^{2}-x-6} d x
$$

4. [] (§7.3) Let $f(t)=\frac{\sqrt{x^{2}-1}}{x}$. Find the average value of $f$ on the interval $[1,7]$.
5. [] (§) The region under the curve $y=\cos ^{2}(x)$ from $0 \leq x \leq \pi$ is rotated about the $x$-axis, find the volume of the resulting solid.
6. (a) [8] Interpret $\int_{1}^{e^{2}} \ln x-\left(\frac{-1}{e(e-1)} x+\frac{e}{e-1}\right) d x$ as the area of a region, by sketching a graph. Hint: $x=e$ and $x=e^{2}$ are good points to plot.
(b) [4] Interpret $\int_{1}^{e^{2}}\left|\ln x-\left(\frac{-1}{e(e-1)} x+\frac{e}{e-1}\right)\right| d x$ as the area of a region.
(c) [5] Evaluate $\int_{1}^{e^{2}}\left|\ln x-\left(\frac{-1}{e(e-1)} x+\frac{e}{e-1}\right)\right| d x$.
7. () A factory worker is trying to push a large package suspended from a track on the ceiling a meter to the right. Conveniently the worker's arm length is 1 meter and she can apply 130 Newtons to do so. However, given her short height she can only apply the force at an angle. Initially she can only push the package up and to the right making an angle of $75^{\circ}$ with the horizontal, but by the end of the 1 meter she has a better angle of $30^{\circ}$ (picture attempted below). Assume the angle varies linearly with the distance that the package travels. How much work does the factory worker do on the object?
distance travelled=1m

8. Technical communication questions"
(a) State the rule for integration by parts. In practice, how do you use it?
(b) How would you evaluate $\int \sin ^{a}(x) \cos ^{b}(x) d x$ if $a$ is odd? What if $b$ is odd?
(c) How would you evaluate $\int \sin ^{a}(x) \cos ^{b}(x) d x$ if $a$ and $b$ are odd?
(d) How would you evaluate $\int \sin ^{a}(x) \cos ^{b}(x) d x$ if $a$ and $b$ are even?
(e) How would you evaluate $\int \tan ^{a}(x) \sec ^{b}(x) d x$ if $b$ is even and $a$ is odd?
(f) How would you evaluate $\int \cot ^{a}(x) \csc ^{b}(x) d x$ if $b$ is even? If $a$ is odd?
(g) Can you integrate any rational function? Justify your answer.
