EXAM 1 | TMath 125 | PRACTICE

TMath 125

EXAM 1 PRACTICE

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let a, b, and c be constants. Assume f and g are continuous.

   T F $\int_a^b cf(x)dx = c\int_a^b f(x)\,dx$

   T F $\int f(x)g(x)dx = \int f(x)\,dx \int g(x)dx$

   T F All continuous functions have derivatives.

   T F All continuous functions have antiderivatives.

   T F $\int_{-1}^{1} \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -2$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. Oil leaked from a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed and values of the rate at two hour time intervals are shown in the table. Find lower and upper estimates for the total amount of oil that leaked out.

t (hours)	0	2	4	6	8	10
$f(t)$ (Liters/hour)	8.7	7.6	6.8	6.2	5.7	5.3

3. Let a be a constant (like 2.5 or something). Find the equation of the line that is tangent to the graph of $y = \ln x$ at $x = e^a$ for some constant a.

$$f(x) = \begin{cases} 2\sin\left(\frac{\pi}{2}x\right) & \text{if } x < -2 \\ -\sqrt{4-x^2}; & \text{if } -2 \le x \le 2 \\ x-2; & \text{if } 2 < x \end{cases}$$

4. Refer to the above definition of $f(x)$ to answer the following questions.

   (a) Carefully graph $f(x)$ from $x = -6$ to $x = 4$.

   (b) Use your above graph to find $\int_{-2}^{4} f(x)\,dx$.

   (c) Give a rough sketch the graph of $\int_0^x f(t)\,dt$ from $x = -6$ to $x = 4$.

5. Find

$\frac{d}{dx}\int_0^{\tan x}\sqrt{1+r^3}\,dr$ $\qquad$ $\left(\int_x^0 e^{\arctan(t)}\,dt\right)'$

6. Evaluate the following.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2\sin x}{1+x^6}\,dx \qquad \int x^3\sqrt{x^2+1}\,dx$$

$$\int \tan(x)\,dx \qquad \int_4^7 \sqrt{2t+1}\,dt$$

7. Consider $y = \sin(2x)$ and $y = \cos(x)$. Find the area of the region bounded by the above between $x = 0$ and $x = \pi$.

8. Consider that area bounded by $y = \ln x$, $y = 1$, $y = 2$, $x = 0$. Find the volume of the solid that results by rotating the above region about the line $x = -1$

9. Kobayashi has won the hot dog-eating world championship six times. Recently he challenged a giant bear to a 3 minute hot dog-eating contest. Kobayashi found that the rate he can eat hot dogs goes down as time goes by and can be modeled by $k(t) = \frac{12}{(t+1)^2} + 24$, were t is measured in minutes. The bear isn't quite as used to the system and seems to start with a slower rate that gets larger and is well modeled by $b(t) = 8t^3 + 20$. Find out how many hot dogs Kobayashi and the bear eat and determine who won the context.

10. A tank has the shape of an inverted circular cone with height 10m and base radius 4m. It is filled with water to a height of 8m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000kg/m^3.)