Key

Exam 1 | **TMath 125** | Spring 2014

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be continuous functions and let a, b, and c be constants.

T F $\frac{\sqrt{x}}{x-5} = \frac{\sqrt{x}}{x} - \frac{\sqrt{x}}{5}$ $\quad \frac{\sqrt{x}}{x} - \frac{\sqrt{x}}{5} = \frac{5\sqrt{x}}{5x} - \frac{x\sqrt{x}}{5x} = \frac{5\sqrt{x} - x\sqrt{x}}{5x}$

T F $\int x\,dx = \int_0^1 x\,dx$ $\quad$ $\int x\,dx$ is a family of functions $\int_0^1 x\,dx$ is a #

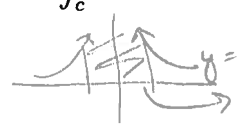
T F $\int \frac{1}{1+x^2}\,dx = \arctan(x) + c$ $\quad$ b/c $\frac{d}{dx}(\arctan(x)+c) = \frac{1}{1+x^2} + 0$

T F All continuous functions have antiderivatives.

T F $\int \frac{f(x)}{g(x)} dx = \frac{\int f(x)\,dx}{\int g(x)\,dx}$ $\quad$ Consider $\int \frac{x^2}{x}\,dx$ vs $\frac{\int x^2\,dx}{\int x\,dx} = \frac{\frac{1}{3}x^3 + d}{\frac{1}{2}x^2 + e}$ $\quad \int x\,dx = \frac{1}{2}x^2 + c$

T F $\int_a^b f(x) + g(x)\,dx = \int_a^c f(x) + g(x)\,dx + \int_c^b f(x) + g(x)\,dx$ $\quad$ property of integrals

T F $\int_{-1}^{1} \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -2$



should be positive area

T F Marginal revenue is similar to the derivative of the revenue function.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [5] You have a friend in Calculus 1 this term who has just learned what a derivative is. Explain ~~how~~ to your friend how the derivatives relates to areas.

start +.5
true +1
+1 FTC II
+1 antiderivative/def
+1 sense
+.5 on topic

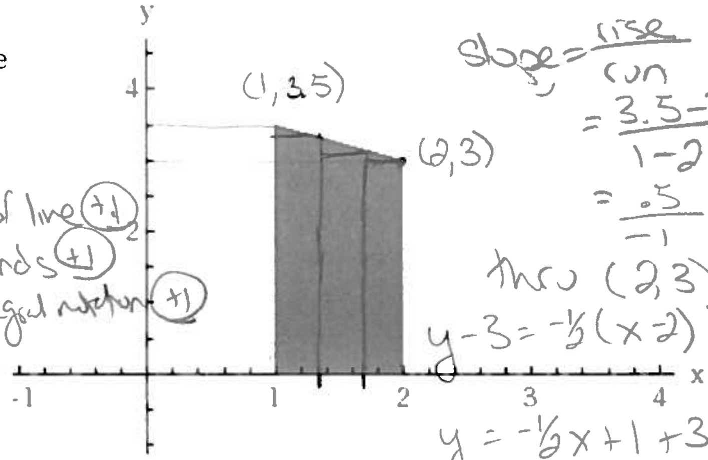
Start with a function f. You learned how to find f'.

Here we'll start with f' and ask what area is trapped between the graph of f', the x-axis, from $x=0$ to $x=2$ maybe.

Well it turns out the area is equal to the original function f with a bit of evaluation. $f(2)-f(0)$ to be exact.

So the area trapped by the derivative & the x-axis on a closed interval $[a,b]$ can be found by evaluating the original function. It's also some

3. Consider the shaded region beneath the straight line shown on the right:



slope $= \frac{\text{rise}}{\text{run}} = \frac{3.5-3}{1-2} = \frac{.5}{-1} = -\frac{1}{2}$

thru (2,3)

$y-3 = -\frac{1}{2}(x-2)$

$y = -\frac{1}{2}x + 1 + 3$

$y = -\frac{1}{2}x + 4$

(a) [3] (def. int. wks #2) Describe the shaded area as a definite integral.

eq of line (+1)
bounds (+1)
integral notation (+1)

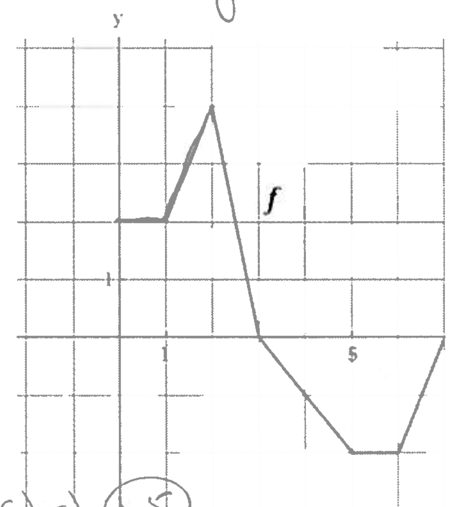
$$\int_1^2 -\frac{1}{2}x + 4 \, dx$$

(b) [3] (WebHW2 #6) Use three rectangles with a base length of $\frac{1}{3}$ to approximate the area. Specify if you are using left-hand, right-hand, or midpoint approximations

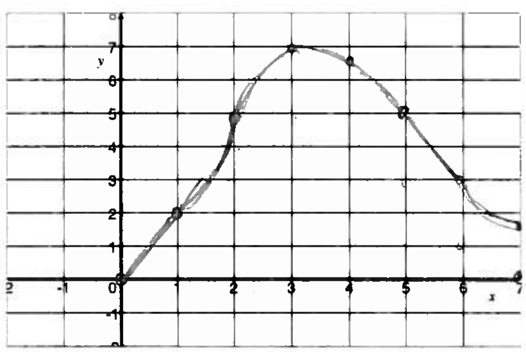
(+.5) { left hand approx   base (+.5)   heights (+1)   3 rectangles (+.5)

start (+.5) $\frac{1}{3} \cdot 3.4 + \frac{1}{3} \cdot 3.2 + \frac{1}{3} \cdot 3$

4. Consider the piecewise-defined velocity function f defined in the graph to the right.



(a) [3] (Quiz1 #2) Sketch the graph of $F(x) = \int_0^x f(x)\,dx$ on the graph below.



start (+.5)
positive area (+1)
no negative area ()
got it (+.5)

(b) [2] (Quiz 2 #2) Find the *total* distance traveled from $t = 0$ to $t = 7$.

(+1) { Total distance so $\int_0^7 |f(x)|\,dx$ (+.5) (+.5) (travel in the negative direction is still distance)

$\int_0^3 f(x)dx + -\int_3^7 f(x)dx = 7 + 5 = 12$

5. [2] (WebHW5 #10) If g is continuous and $\int_{-3}^{7} \frac{1}{2} g(x)\,dx = 4$, find $\int_0^5 g(2x-3)\,dx$.

$$\int_0^5 g(2x-3)dx = \int_{2\cdot 0-3}^{2\cdot 5-3} g(u)\,\frac{1}{2}du = \int_{-3}^{7} \frac{1}{2} g(u)du = 4$$

(+.5)   (+.5)

let $u = 2x-3$ } (+.5)

$du = 2dx$ } (+.5)

$\frac{1}{2}du = dx$

6. Find the following or explain why it does not exist.

[2] (Substitution Wks #1)

$$\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx = \int \sin(\sqrt{x}) \cdot \frac{1}{\sqrt{x}}\, dx$$

sub (+.5)

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2} x^{-1/2} dx = \frac{1}{2\sqrt{x}} dx$

$\Rightarrow 2du = \frac{1}{\sqrt{x}} dx$

$= \int \sin(u) \cdot 2du$

$= 2\int \sin(u)du$ (+.5)

$= -2\cos u + c$

$= -2\cos(\sqrt{x}) + c$ (+.5)

(+.5) answer is in x

Ck: $(-2\cos(\sqrt{x}) + c)' = +2\sin\sqrt{x}\,(x^{-1/2}\tfrac{1}{2}) + 0$ ✓

[3] (WebHW4 #9)

$$\int_0^1 x^{10} + 10^x\, dx$$ (+.5) (+.5) notation (+.5)

$\int x^{10} + 10^x dx = \frac{1}{11}x^{11} + \frac{1}{\ln 10}10^x + c$

Ck: $\left(\frac{1}{11}x^{11} + \frac{1}{\ln 10}\cdot 10^x + c\right)'$

$= \frac{11}{11}x^{10} + \frac{1}{\cancel{\ln 10}}10^x \cancel{\ln 10} + 0$

(+1) FTC II

$\int_0^1 x^{10} + 10^x dx = \frac{1}{11}x^{11} + \frac{1}{\ln 10}10^x \Big]_0^1$

$= \left(\frac{1}{11}(1) + \frac{1}{\ln 10}\cdot 10\right) - \left(\frac{1}{11}\cdot 0 + \frac{1}{\ln 10}\cdot 1\right)$

plug in (+.5)

$= \frac{1}{11} + \frac{10}{\ln 10} - \frac{1}{\ln 10} = \frac{1}{11} + \frac{9}{\ln 10}$

$\approx .0909 + \frac{9}{2.303} = 3.999$

[3] (WebHW3 #10)

$$\frac{d}{dx}\left(\int_9^{\tan(x)} \sqrt{5t + \sqrt{t}}\, dt\right)$$

FTC I (+.5) and the Chain rule (+.5)

$= \sqrt{5\tan(x) + \sqrt{\tan(x)}}\,(\tan(x))'$

$= \underbrace{\sqrt{5\tan(x) + \sqrt{\tan(x)}}}_{(+1)}\ \underbrace{(\sec^2(x))}_{(+.5)}$

answer is in x (+.5)

[2] (§5.5 #68)

$$\int \frac{2x+8}{x^2+8x+16}\, dx$$ typo

$\int \frac{2\cancel{(x+4)}}{\cancel{(x+4)}(x+4)} dx$

$= \int \frac{2}{x+4} dx$

$u = x+4$, $du = dx$ (+.5)

$= \int \frac{2}{u} du$ (+.5)

$= 2\ln|u| + c$ (+.5)

$= 2\ln|x+4| + c$

$= \ln|x+4|^2 + c$

$= \ln|x^2+8x+16| + c$

or $u = x^2+8x+16$, $du = (2x+8)dx$

$\int \frac{1}{u}(2x+8)dx$

$= \int \frac{1}{u} du$

$= \ln|u| + c$

$= \ln|x^2+8x+16| + c$

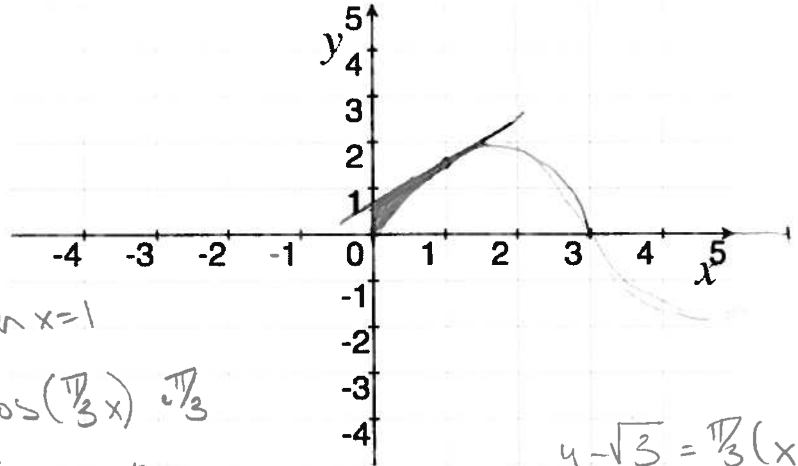
all are ok

answer is in x (+.5)

7. (§6.1 #50) Consider the region bounded by the curve $f(x) = 2\sin\left(\frac{\pi}{3}x\right)$, the line tangent to f when $x = 1$, and the y-axis.

(a) [1] Carefully draw the region described above.

(b) [2] Find the equation of the line tangent to f when $x = 1$.



Looking for $y - y_1 = m(x - x_1)$

(+.5) m = slope of line tangent to f when $x=1$

$= f'(1)$ (+.5) $f'(x) = 2\cos(\frac{\pi}{3}x)\cdot\frac{\pi}{3}$

$= 2\cdot\frac{\pi}{3}\cdot\cos(\frac{\pi}{3}\cdot 1)$

$= 2\cdot\frac{\pi}{3}(\frac{1}{2}) = \frac{\pi}{3}$ (+.5)

(+.5) passes thru $(1, 2\sin(\frac{\pi}{3}(1))) = (1, 2\cdot\frac{\sqrt{3}}{2}) = (1, \sqrt{3})$

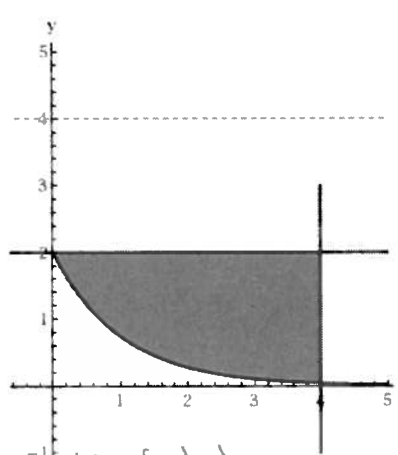
$y - \sqrt{3} = \frac{\pi}{3}(x - 1)$ or $y = \frac{\pi}{3}x - \frac{\pi}{3} + \sqrt{3}$ $\approx 1.047x + .685$

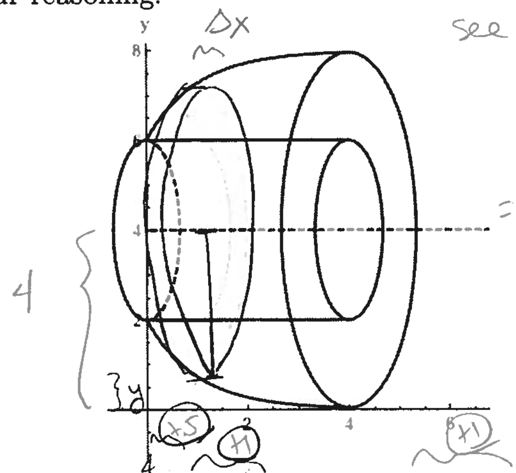
(c) [2] Set up the integral *but do not find/integrate* that corresponds to the area sketched in part (a).

$$\int_0^1 \left(\frac{\pi}{3}x - \frac{\pi}{3} + \sqrt{3}\right) - 2\sin\left(\frac{\pi}{3}x\right)dx$$

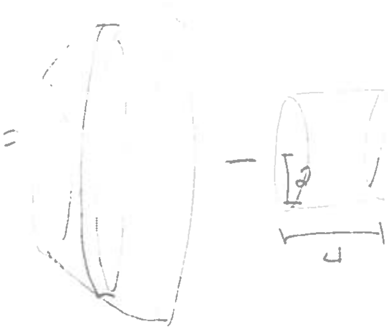
(+.5) subtracting (+.5) order is right (+.5) wrt x (+.5)

8. (§6.2 #12) [4] Consider the region shown below on the left where the curve is defined by $y = 2e^{-x}$ and the dotted line is the axis about which the region will be rotated to produce the volume shown on the right. Set up the definite integral to find the volume of the object shown below on the left. You do *not* need to integrate but be sure to show your work so I can follow your reasoning!





see cross sections (+.5)



(+.5) (+1) (+1)

Vol of Solid − Vol of hole

Sum of vol of pieces $- 4\cdot\pi\cdot 2^2$

Sum $\pi(4-y)^2\Delta x$

$\lim_{\#\text{ of pieces}\to\infty}$ Sum $\pi(4-y)^2\Delta x\Big] - 4\pi\cdot 4$

$$\int_0^4 \pi(4-y)^2 dx - 16\pi$$

$$\int_0^4 \pi(4-2e^{-x})^2 dx - 16\pi$$

(+.5) in terms of x (+.5)

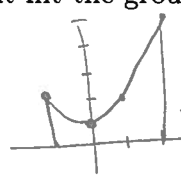
9. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (Word Problem Wks #3) The gravity (acceleration imposed on an object by the planet) on mars is $3.69\frac{m}{s^2}$. Let's consider what could happen if NASA's next Mars explorer has "boosters" which will to let the explorer fly up with a maximum upwards velocity of 5 meters per second.

i. [2] If the new explorer jumped off the edge of a cliff 7250m with a maximum upwards velocity, find a function to describe the velocity t seconds after the explorer jumps off the edge.

ii. [3] What speed would the explorer be going when it hit the ground?

(b) (Word Problem Wks #7) Consider $f(x) = 1 + x^2$



note f(x) evaluates to the #'s given in i) but the average is different?

i. [1] Find the average of 2, 1, 2, and 5.

ii. [2] Find the average of $f(x)$ from $x = -1$ to $x = 2$.

iii. [2] If we were to "deform" the area $\int_{-1}^{2} f(x)\,dx$ into a box 3 units (or $2-(-1)$ units) wide and l units long. What would t be?

a) Recall

$\text{Position} \xrightarrow{d/dt} \text{velocity} \xrightarrow{d/dt} \text{acceleration}$ ($\int$ back)

i) Since acceleration, $a(t) = -3.69$

$v(t) = \int a(t)\,dt$

$= -3.69t + c$ (+1)

We know at $t=0$ $v = 5\ m/s$ signs (+.5)

So $5 = -3.69 \cdot 0 + c$ c=5 (+.5)

$\Rightarrow c = 5$

So $v(t) = -3.69t + 5$

ii) ie find velocity when position $p(t) = 0$ (+.5)

$p(t) = \frac{-3.69t^2}{2} + 5t + c$ (+1)

Since $p(0) = 7250$ $c = 7250$ (+.5)

$p(t) = \frac{-3.69}{2}t^2 + 5t + 7250$

find zeros $t = \frac{-5 \pm \sqrt{25 - 4\left(\frac{-3.69}{2}\right)(7250)}}{2 \cdot \frac{-3.69}{2}}$ (+.5)

$\Rightarrow t =$ So $v(\quad) =$ (+.5)

b) i) (+.5) $\frac{2+1+2+5}{4} = \frac{10}{4} = \frac{5}{2} = 2.5$

ii) $\underbrace{\frac{1}{2-1}}_{(+.5)} \underbrace{\int_{-1}^{2}}_{(+.5)} 1 + x^2\,dx = \frac{1}{3}\underbrace{\left[x + \frac{1}{3}x^3\right]_{-1}^{2}}_{(+.5)}$

$= \frac{1}{3}\left[\left(2 + \frac{8}{3}\right) - \left(-1 - \frac{1}{3}\right)\right] = \frac{1}{3}\left[\frac{14}{3} + \frac{4}{3}\right]$

$= \frac{1}{3}\left(\frac{18}{3}\right) = \frac{1}{3} \cdot 6 = 2$ plug in (+.5)

iii) from part ii we know

$\frac{1}{3}\int_{-1}^{2} 1 + x^2\,dx = 2$

or $\underbrace{\int_{-1}^{2} 1 + x^2\,dx}_{\text{area trapped between } 1+x^2 = y, \text{ the x axis from } x=-1 \text{ to } x=2} = \underbrace{2 \cdot 3}_{\text{the area of a box 3 units wide and 2 units long}}$ (+.5) (+.5)

So 2 (+1)

~~13~~   19
~~13~~   26
~~10~~   5
~~10~~   ---
~~5~~   50