Using Theorem presented in class where we change the limits

Using the FTC II without changing the limits

2. Find:

$$\int_{-1}^{3} e^{5x}\,dx = \int_{5(-1)}^{5(3)} e^{u}\tfrac{1}{5}\,du$$

$u=5x$
$du=5dx$
$\Rightarrow \tfrac{1}{5}du=dx$

$$= \int_{-5}^{15} \tfrac{1}{5}e^{u}\,du$$
$$= \tfrac{1}{5}\int_{-5}^{15} e^{u}\,du$$
$$\overset{FTC\ II}{=} \tfrac{1}{5}\left[e^{u}\right]_{-5}^{15}$$
$$= \tfrac{1}{5}\left[e^{15}-e^{-5}\right]$$

$$\int_{4}^{9} (x+1)(2x+x^2)^{\frac{5}{2}}\,dx$$

$\int (x+1)(2x+x^2)^{5/2}dx = \frac{1}{7}(2x+x^2)^{7/2}+c$ from other side of WKS so

$$\int_4^9 (x+1)(2x+x^2)^{5/2}dx = \tfrac{1}{7}(2x+x^2)^{7/2}\Big]_4^9$$
$$= \tfrac{1}{7}(2\cdot 9+9^2)^{7/2} - \tfrac{1}{7}(2\cdot 4+4^2)^{7/2}$$
$$= \tfrac{1}{7}(99)^{7/2} - \tfrac{1}{7}(24)^{7/2}$$

$$\int_{1}^{4} \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = \int_1^4 \sin\sqrt{x}\,\frac{1}{\sqrt{x}}\,dx$$

$u=\sqrt{x}$
$du=\frac{-1}{2\sqrt{x}}dx$
$\Rightarrow -2du=\frac{1}{\sqrt{x}}dx$

$$= \int_{\sqrt{1}}^{\sqrt{4}} \sin(u)\,(-2)\,du$$
$$= -2\int_1^2 \sin(u)\,du$$
$$= +2\cos(u)\Big]_1^2$$
$$= 2\cos(2) - 2\cos(1)$$

$$\int_{1}^{4} \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$$

$u=1+\frac{1}{x}$
$du=\frac{-1}{x^2}dx$
$\Rightarrow -du=\frac{1}{x^2}dx$

$$\int \frac{1}{x^2}\sqrt{1+\tfrac{1}{x}}\,dx$$
$$= \int \sqrt{u}\,\frac{1}{x^2}dx$$
$$= \int \sqrt{u}\,(-1)\,du$$
$$= -\int \sqrt{u}\,du$$
$$= -\tfrac{2}{3}u^{3/2}+c$$
$$= -\tfrac{2}{3}\left(1+\tfrac{1}{x}\right)^{3/2}+c$$

Check: $\left(-\frac{2}{3}(1+\frac{1}{x})^{3/2}+c\right)'$
$= +\frac{2}{3}\cdot\frac{3}{2}(1+\frac{1}{x})^{1/2}\cdot\frac{+1}{x^2}+0$ ✓

$$\int_1^4 \frac{1}{x^2}\sqrt{1+\tfrac{1}{x}}\,dx = \frac{-2}{3}\left(1+\frac{1}{x}\right)^{3/2}\Big]_1^4$$
$$= -\tfrac{2}{3}\left(1+\tfrac{1}{4}\right)^{3/2} - \tfrac{-2}{3}\left(1+\tfrac{1}{1}\right)^{3/2}$$
$$= -\tfrac{2}{3}\cdot\left(\tfrac{5}{4}\right)^{3/2} + \tfrac{2}{3}(2)^{3/2}$$

$$\int_{0}^{1} \frac{1}{(1+\sqrt{x})^4}\,dx$$

$u=1+\sqrt{x}$
$\Rightarrow u-1=\sqrt{x}$
$du=\frac{1}{2\sqrt{x}}dx$
$\Rightarrow 2\sqrt{x}\,du=dx$

$$= \int_{1+\sqrt{0}}^{1+\sqrt{1}} \frac{1}{u^4}\cdot 2\sqrt{x}\,du$$
$$= \int^2 \frac{2}{u^4}\sqrt{x}\,du$$
$$= \int_1^2 \frac{2}{u^4}(u-1)\,du$$
$$= 2\int_1^2 \frac{1}{u^4}(u-1)\,du = 2\int_1^2 \frac{u}{u^4}-\frac{1}{u^4}\,du$$
$$= 2\int_1^2 \frac{1}{u^3}-\frac{1}{u^4}\,du = 2\left(\frac{-1}{2}u^{-2}+\frac{1}{3}u^{-3}\right)\Big]_1^2 = -u^{-2}+\frac{2}{3}u^{-3}\Big]_1^2 = \left(-\frac{1}{2^2}+\frac{2}{3\cdot 2^3}\right)-\left(-\frac{1}{1^2}+\frac{2}{3}\frac{1}{1^3}\right)$$