2. Find:

$$\int_0^3 x\sqrt{x+1}\,dx$$

~~Know~~ ~~alg~~ sub?

$u = x+1 \Rightarrow u-1 = x$

$du = dx$ Substitution?

$\int x\sqrt{x+1}\,dx = \int (u-1)u^{1/2}\,du$

$= \int u^{3/2} - u^{1/2}\,du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + c$

$= \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + c$

fond antider.

$\int_0^3 x\sqrt{x+1}\,dx \underset{FTC}{=} \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}\Big]_0^3$

$= \left[\frac{2}{5}(4)^{5/2} - \frac{2}{3}(4)^{3/2}\right] - \left[\frac{2}{5}1 - \frac{2}{3}\right]$

$$\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \theta^3\cos(\theta^2)\,dx$$

~~Know~~ ~~alg~~ sub?

$u = \theta^2$

$du = \theta\,d\theta$

$\int (\cos(u))\,\theta^2\,\theta\,d\theta = \int (\cos w)\,w\,dw = \int w\cos w\,dw$

~~Know?~~ ~~alg~~ ~~sub~~ IP

$u = w$ $\quad v = \sin w$

$du = dw$ $\quad dv = \cos w\,dw$

$\int w\cos w\,dw = w\sin w - \int \sin w\,dw$

$= w\sin w + \cos w + c$

$= \theta^2\sin(\theta^2) + \cos\theta^2 + c$

$\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3\cos(\theta^2)\,d\theta \underset{FTCII}{=} \theta^2\sin(\theta^2) + \cos\theta^2\Big]_{\sqrt{\pi/2}}^{\sqrt{\pi}}$

$= (\pi\sin\pi + \cos\pi) - (\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}) = -1 - \frac{\pi}{2}$

$$\int_1^2 \frac{\ln(y)}{\sqrt{y}}\,dy$$

~~Know~~ ~~alg~~ sub? IP

$u = \ln y$ $\quad v = 2y^{1/2}$

$du = \frac{1}{y}dy$ $\quad dv = \frac{1}{\sqrt{y}}dy = y^{-1/2}dy$

$\int_1^2 \frac{\ln y}{\sqrt{y}}\,dy = \left[(\ln y)2y^{1/2}\right]_1^2 - \int_1^2 2y^{1/2}\frac{1}{y}\,dy$

$= 2y^{1/2}\ln y\Big]_1^2 - 2\int_1^2 y^{-1/2}\,dy = 2y^{1/2}\ln y - 2\cdot 2y^{1/2}\Big]_1^2$

$= (2\sqrt{2}\ln 2 - 4\sqrt{2}) - (2\ln 1 - 4\cdot 1)$

$$\int_0^1 5^t\sin(5^t)\,dx$$

~~Know?~~ ~~alg~~ sub?

$u = 5^t$

$du = 5^t\ln 5\,dt \Rightarrow \frac{1}{\ln 5}du = 5^t dt$

$\int_0^1 5^t\sin 5^t\,dt = \int_1^5 \sin(u)\frac{1}{\ln 5}\,du$

$= \frac{1}{\ln 5}\int_1^5 \sin u\,du = \frac{1}{(\ln 5)}(-\cos u\Big]_1^5$

$= \frac{1}{\ln 5}[-\cos 5 - -\cos 1]$

$= \frac{1}{\ln 5}[\cos 1 - \cos 5]$

3. (§7.1 #66) A rocket accelerates by burning its onboard fuel, so the mass of the rocket decreases with time. Suppose the initial mass of the rocket at lift off (including its fuel) is m, the fuel is consumed at a rate r, and the exhaust gases are ejected with constant velocity v (relative to the rocket). A model for the velocity of the rocket at time t is given by the equation

$$v(t) = -gm - v_c ln\frac{m - rt}{m}$$

where g is the acceleration due to gravity and t is not too large. If the rocket is on earth, $m = 30,000$kg, $r = 160\frac{\text{kg}}{\text{s}}$, and $v_c = 3000\frac{\text{m}}{\text{s}}$, find the height of the rocket one minute after liftoff.

62 ex total change in height $= \int_0^{60} v(t)\,dt$

$$\int_0^{60} -gt - v_e \ln\left(\frac{m-rt}{m}\right)dt = \int_0^{60} -9.8t - 3000\ln\left(\frac{30{,}000-160t}{30{,}000}\right)dt$$

$$= -9.8\frac{t^2}{2}\Big|_0^{60} - 3000\int_0 \ln\left(\frac{m-rt}{m}\right)dt$$

$u = \frac{m-rt}{m}$ $\quad du = \frac{-r}{m}dt$

$\frac{-m}{r}du = dt$

$$= -17640 - 3000\int^{\frac{m-r60}{m}} (\ln u)\left(\frac{-m}{r}\right)du$$

$$= -17640 - 300\cancel{0}\left(\frac{-30{,}000}{16\cancel{0}}\right)\int_1^{\frac{m-r60}{m}} \ln u\,du$$

$$= -17640 + \frac{300\cdot 30{,}000}{16}\left(u\ln u - \int u\cdot \tfrac{1}{u}\,du\right)_1^{\frac{m-r60}{m}}$$

$u = \ln u \quad v = u$

$du = 1/u\,du \quad dv = du$

$$= -17640 + \frac{75\cdot 30{,}000}{4}\left(u\ln u - u\right)_1^{\frac{m-r60}{m}}$$

$$= -17640 + 75\cdot 7500\left(\frac{m-r60}{m}\left(\ln\frac{m-r60}{m} - 1\right) - (1\ln 1 - 1)\right)$$

$$= -17640 + 75\cdot 7500\left(\frac{30{,}000-160\cdot 60}{30{,}000}\left(\ln\frac{30{,}000-160\cdot 60}{30{,}000} - 1\right) - (0-1)\right)$$

$(-1.38566$

$= 14844$ m