#2

Simpson's Rule:

Newton - Cotes formula for approximating the integral of a function f using quadratic polynomials (parabolic arcs instead of straight line segments)

Saying its Simpsons Rule

$$\int_a^b f(x)\,dx = S_N = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + f(x_N)\right)$$

how many segments

$$\int_0^6 f(x)\,dx = S_{12} = \frac{1}{3}\cdot\frac{1}{2}\Big(f(0) + 4f(.5) + 2f(1) + 4f(1.5) + 2f(2) + 4f(2.5) + 2f(3) + 4f(3.5) + 2f(4) + 4f(4.5) + 2f(5) + 4f(5.5) + f(6)\Big)$$

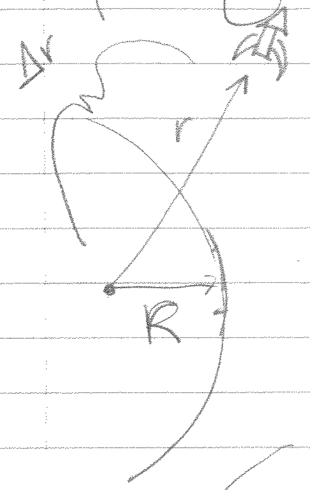
These points using data provided

$$\frac{1}{6}\big(1814 + 6940 + 3372 + 6584 + 3274 + 6436 + 3208 + 6444 + 3242 + 6664 + 3490 + 7544 + 2052\big)$$

$$= \frac{1}{6}(61064)$$

$S_{12} = 10{,}177.\overline{33}$ = energy used during that time period.

# #4




$$F = G\frac{m_1 m_2}{r^2}$$

$m_1$ = mass of Earth = $5.98 \times 10^{24}$ kg

$m_2$ = mass of rocket = 1000 kg

$R = 6.37 \times 10^{6}$ m

$G = 6.67 \times 10^{-11}$ $Nm^2/kg^2$

$$\Delta W = Fd = F\Delta r = \left(G\frac{m_1 m_2}{r^2}\right)\Delta r$$

• Since the gravitational force is proportional to the inverse square of the radius, it diminishes as altitude increases. We need to sum the products of the incremental forces & distances as rocket rises. $\rightarrow \Delta r \rightarrow dr$

$$W = Gm_1m_2\int \frac{1}{r^2}\,dr$$ Infinitesimal work in terms of r

$$W = \lim_{t\to\infty} Gm_1m_2\left(-\frac{1}{r}\right)_R^t = Gm_1m_2\left(-\frac{1}{\infty} - \frac{-1}{6.37\times10^{6}\,m}\right)$$

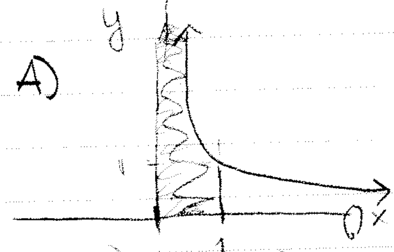
$$= (6.67\times10^{-11}\,Nm^2/kg^2)(5.98\times10^{24}\,kg)(1000\,kg)\left(0 + 1.57\times10^{-7}\,m\right)$$

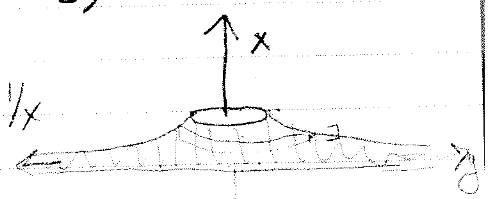
$\Delta W = 6.26 \times 10^{10}$ Joules or 62.6 Gigajoules

• The work "W", required to completely eliminate the gravitational pull on the rocket is the sum of all those infinitesimal forces

#9 Email to Ruth

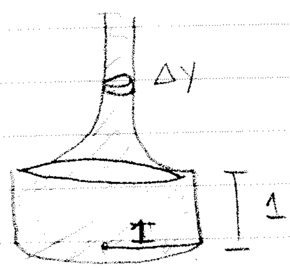
$f(x) = \frac{1}{x}$

A) 


B) 


A area:
$\int_\phi^1 1/x\,dx$

$\lim_{t\to\phi^+} \ln x\Big]_t^1 = \lim_{t\to\phi^+} \underbrace{\ln(1)}_{\phi} - \underbrace{\ln(t)}_{-\infty} = \infty$

c) 


B Volume (around x)

$\int_\phi^1 \pi(1/x)^2\,dx$    $\pi(1)^2(1)$ $\pi$

$\pi\int_\phi^1 1/x^2\,dx \Rightarrow -\pi/\cancel{x}\Big]_\phi^1$

$\lim_{t\to\phi^+} -\pi/x\Big]_t^1 = \lim_{t\to\phi^+} -\pi/1 + -\pi/t$    $-\pi + \infty = \cdot$    Diverged

C Volume (around y)    $\int_\phi^\infty \pi(1/y^2)\,dy = \int_\phi^1 \pi\, 1/y^2\,dy + \int_1^\infty \pi\, 1/y^2\,dy$

$\pi(1)^2(1) = V_{Base}$

$\pi + \pi\int_1^\infty (1/y)^2\,dy$

$\lim_{t\to\infty}\int_1^t 1/y^2\,dy = \lim_{t\to\infty} -\pi/x\Big|_1^t = \lim_{t\to\infty}(-\pi/t - -\pi/1)$

$\phi + \pi = \pi$

$\pi + \pi = 2\pi$

$\lim_{t\to 0^+} \pi/t = \pi \lim_{t\to 0^+} 1/t = \infty$