Word Problem Practice take 3

1. (Ch. 9) Newton's Lw of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large. Find the rule of a function $H(t)$ that obeys Newton's Law of Cooling where $H(t)$ gives the current temperature of a freshly poured cup of coffee (with temperature 190°F) in a room where the temperature is 70°F. You should have trouble finding the function H, what other information do you need to find H exactly?

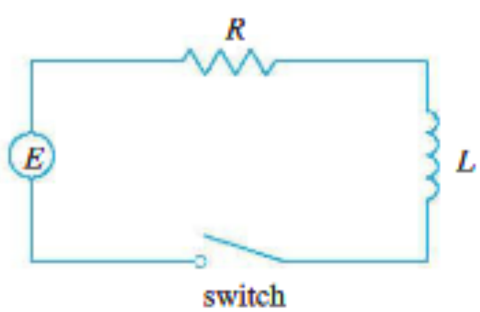
2. (Ch 7) The table (supplied by San Diego Gas and Electric) gives the power consumption P in megawatts in San Diego County from midnight to 6:00am on a day in December. Use Simpson's Rule (and the fact that power is the derivative of energy) to estimate the energy used during that time period.

t	P	t	P
0:00	1814	3:30	1611
0:30	1735	4:00	1621
1:00	1686	4:30	1666
1:30	1646	5:00	1745
2:00	1637	5:30	1886
2:30	1609	6:00	2052
3:00	1604		

3. (Ch. 9) Psychologists interested in learning theory study learning curves. A learning curve is the graph of a function $P(t)$, the performance of someone learning skill as a function of the training time t. The derivative dP/dt represents the rate at which performances improves and is believed to be proportional to the difference between the maximum level of performance of which the learner is capable (call it M) and the actual performance (P). Create and then solve a differential equation involving the function P.

4. (Ch. 7) Newton's Law of Gravitation states that two bodies with masses m_1 and m_2 attract each other with a force $F = G\frac{m_1 m_2}{r^r}$ where r is the distance between the bodies and G is the gravitational constant. Compute the work required to launch a 1000 kg satellite out of earth's gravitational field. Assume the earth's mass is 5.98×10^{24}kg and is concentrated at its center. The radius of the earth is about 6.37×10^6 m and let $G = 6.67 \times 10^{-11}$ Nm^2/kg^2.

5. Consider the electric circuit shown to the right. The circle is a battery supplying a constant 60 volts (V) and a current $I(t)$ amperes (A) at time t. The circuit also contains a resistor with resistance 12 ohms (Ω) and an inductor with inductance 4 henries (H).



Ohm's Law gives the drop in voltage due to the resistor as RI. The voltage drop due to the inductor is $L(dI/dg)$. Once of Kirchhoff's laws says that the sum of the voltage drop is equal to the supplied voltage $E(t)$.

Piece the above physics laws together to form a differential equation

6. (Ch. 9) Experiments show that the reaction $H_2 + Br_2 \rightarrow 2HBr$ satisfies the rate law

$$\frac{d[\mathrm{HBr}]}{dt} = k[\mathrm{H}][\mathrm{Br}]^{\frac{1}{2}}$$

where [H] is the concentrations of a moles of H per L,
[Br] is the concentrations of b moles of Br per L, and
[HBr] is the concentrations of x moles of HBr per L. Thus the above differential equation can be rewritten as

$$\frac{dx}{dt} = k(a-x)(b-x)^{\frac{1}{2}}.$$

Find x as a function of t in the case where $a = b$ and assume $x(0) = 0$.

7. (Ch 9) A glucose solution is administered intravenously into the bloodstream at a constant rate t. As the glucose is added, it is converted into other substances and removed from the bloodstream at a rate that is proportional to the concentration at that time. Let $C(t)$ be the concentration of glucose solution in the bloodstream and assume $C(0) = 0$. Find the rule of $C(t)$ by solving a differential equation and describe the behavior as $t \to \infty$.

8. (Ch 9) Justify the logistic differential equation as a model for population growth:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

Use the logistic model to find the population P of fish in a lake at time t. Biologists first stocked the late with 400 fish and estimated the carrying capacity to be 10,000. Biologists returned in a year and found that the population had already tripled.

9. (Ch 7) Consider the region trapped between $f(x) = \frac{1}{x}$, the x-axis, and from $x = 0$ to $x = 1$. What is the area of the region? What would it's volume be if it was revolved about the x-axis? What would it's volume be if it was revolved about the y-axis?