Word Problem Practice take 2

1. (§7.3) Recall that a circle is the collection of all (x, y) points a fixed distance R from a point (h, k). Thus the formula for a circle comes from the Pythagorean Theorem: $(x-h)^2 + (y-k)^2 = r^2$. Use this and calculus to prove the area of a circle with radius R is πr^2.

2. (§6.4) Explain the relationship between work (Joules J) and force (Newtons N) using areas and calculus.

   Use the relationship described above to solve the following: A force of 40N is required to hold a spring that has been stretched from its natural length of 10cm to a length of 15cm. How much work is done in stretching the spring from 15cm to 18cm? Make use of Hooke's law that says the force required to maintain a spring stretched x units beyond its natural length is $F(x) = kx$ for a spring constant k.

3. (§7.1 #66) A rocket accelerates by burning its onboard fuel, so the mass of the rocket decreases with time. Suppose the initial mass of the rocket at lift off (including its fuel) is m, the fuel is consumed at a rate r, and the exhaust gases are ejected with constant velocity v_c (relative to the rocket). A model for the velocity of the rocket at time t is given by the equation

$$v(t) = -gt - v_c ln \frac{m - rt}{m}$$

   where g is the acceleration due to gravity and t is not too large. If the rocket is on earth, $m = 30,000$kg, $r = 160\frac{\text{kg}}{\text{s}}$, and $v_c = 3000\frac{\text{m}}{\text{s}}$, find the height of the rocket one minute after liftoff.

4. Explain the mean value theorem and use it to compute the average value of:

   (a) (§7.1) $f(x) = x \sec^2(x)$ on the interval $[0, \frac{\pi}{4}]$, and

   (b) (§7.4) $g(x) = \frac{12x}{x^2+x-2}$ on the interval $[2, 5]$.

5. (§7.4) One method to slow the growth of an insect population without using pesticides is to introduce a number of sterile males that mate with fertile females but produce no offspring. If P represents the number of female insects in a population, S the number of sterile males introduced each generation, and r the population's natural growth rate, then the female population is related to time t by

$$t = \int \frac{P + S}{P[(r - 1)P - S]} dP.$$

   Suppose an insect population with 10,000 females grows at a rate of $r = 0.10$ and 900 sterile males are added. Evaluate the inegral to give an equation relating the female population to time. Note, you do not need to explicitly solve for P.

6. (§7.3) A charged rod of length L produces an electric field at point $P(a, b)$ given by

$$E(P) = \int_{-a}^{L-a} \frac{\lambda b}{4\pi\epsilon_0 (x^2 + b^2)^{\frac{3}{2}} dx}$$

where λ is the charge density per unit length on the rod and ϵ_0 is the free space permittivity. Evaluate the integral to determine an expression for the electric field $E(P)$.

7. (§7.2) Housefold electricity is supplied in the form of alternating current that varies from 155V to -155V with a freqency of 60 cycles per second (Hz). The voltages is thus given by the equation

$$E(t) = 155 \sin(120\pi t)$$

where t is the time in seconds. Voltmeters read the RMS (root-mean-square) voltage, which is the square root of the average value of $[E(t)]^2$ over one cycle.

(a) Calculate the RMS voltage of household current.

(b) Many electric stoves require an RMS voltage of 220V. Find the corresponding amplitude A needed for the voltage $E(t) = A\sin(120\pi t)$.