Word Problem Practice

1. (§6.4) Explain the relationship between work (Joules J) and force (Newtons N) using areas and calculus.

   Use the relationship described above to solve the following: A force of 40N is required to hold a spring that has been stretched from its natural length of 10cm to a length of 15cm. How much work is done in stretching the spring from 15cm to 18cm? Make use of Hooke's law that says the force required to maintain a spring stretched x units beyond its natural length is $F(x) = kx$ for a spring constant k.

2. (§6.5) Explain the mean value theorem and use it to compute the average velocity v of blood in a blood vessel. Use

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

   where R is the radius of the blood vessel, l is the length, η is the viscosity of blood, P is the pressure difference between the ends of the blood vessel, and r is the distance from the central axis.

3. (§4.9) Describe the relationship between position, velocity and acceleration using differential and integral calculus.

   The gravity (acceleration imposed on an object by the planet) on mars is $3.69\frac{\text{m}}{\text{s}^2}$. If NASA accidentally steered Spirit off a cliff 7250m high (yes, they get that deep), find a function to describe the velocity t seconds after Spirit falls off the edge. What speed would Sprit be going when it hit the ground?

4. (§6.1) If the birth rate of a population is $b(t) = 2200e^{0.024t}$ people per year and the death rate is $d(t) = 1460e^{0.018t}$ people per year, find the area between these curves for $0 \leq t \leq 10$. What does the area represent?

5. (§6.4) Explain the relationship between work (Joules J) and force (Newton N) using areas.

   Use the relationship to solve the following: A 200lb cable is 100 ft long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building? Note: there are two very different ways this would be done.

6. (§5.5) Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end takes about 5 seconds. The macimum rate of airflow into the lungs is about $.5\frac{\text{L}}{\text{s}}$. Thus the function $\frac{1}{2}\sin\left(\frac{2\pi t}{5}\right)$ is a reasonable model for the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time t.

7. (§6.5) Explain the mean value theorem (MVT). Use the MVS and the model for the lung described in #7 and use it to compute the average volume of inhaled air in the lungs during one respiratory cycle.

8. Explain what marginal cost, marginal revenue, and marginal profit is.

   A company produces widgets and find that it has some economies of scale. That is, each additional widget produced is slightly cheaper to produce and on average costs less. Say the company estimates that the marginal cost for producing x items is $20e^{-.125x}+2$ dollars but the fixed cost (producing no widgets is \$100). What is the cost of producing 50 widgets?

   If widgets are sold for \$9.99 each, find the marginal revenue of the company and determine how many widgets they should make to maximize profits.

9. §6.2 #50

10. Consider Ohm's law for capacitors: $i = C\dfrac{dv}{dt}$, umm... I ran out of time and didn't get to cook this one. I'd let you try to come up with one, get approval by me, and then present it to the class.