TMATH 125 Quiz 4 Key

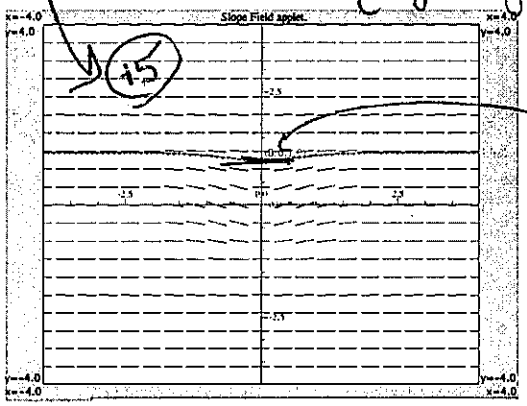
Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [3] (dif eq wks #2) Match the differential equations with the solutions graphs. Briefly *justify* your choice.

(a) $y' = xe^{-x^2-y^2}$

if $y'=0$ then $0 = xe^{-x^2-y^2} = x e^{-(x^2+y^2)}$

$= x / e^{x^2+y^2}$ only if $x=0$



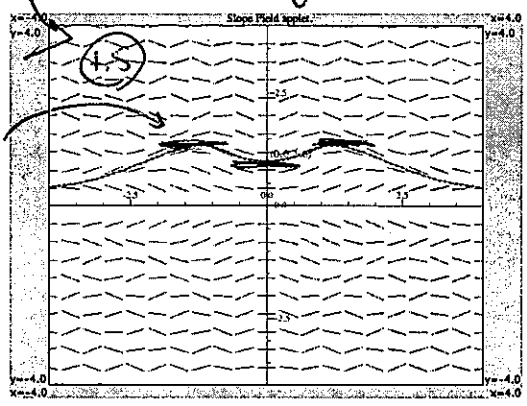
(+5)

(b) $y' = \sin(xy)\cos(xy)$

$(0,0) \Rightarrow y'=0$

$(\sqrt{\pi}, \sqrt{\pi}) \Rightarrow y' = \sin\pi\cos\pi = 0$

zeros at more than once



(+5)

$y'=0$

justify (+1)

sense (+5)

y' = slope of tang. line (+5)

2. [3] (§9.3 #13) Find the solution of the differential equation that satisfies the given initial condition:

$$\frac{du}{dt} = \frac{2t+\sec^2(t)}{2u} \qquad u(0) = -5$$

$2u \frac{du}{dt} = 2t + \sec^2(t)$

$\Rightarrow 2u\,du = [2t+\sec^2(t)]\,dt$

$\Rightarrow \int 2u\,du = \int 2t + \sec^2(t)\,dt$

$2\frac{1}{2}u^2 = 2\frac{1}{2}t^2 + \tan(t) + c$

(+5) (+5) (+5) (+5)

$u^2 = t^2 + \tan(t) + c$

(+1) Since $u(0) = -5$

$(-5)^2 = (0)^2 + \tan(0) + c$

$\Rightarrow 25 = c$

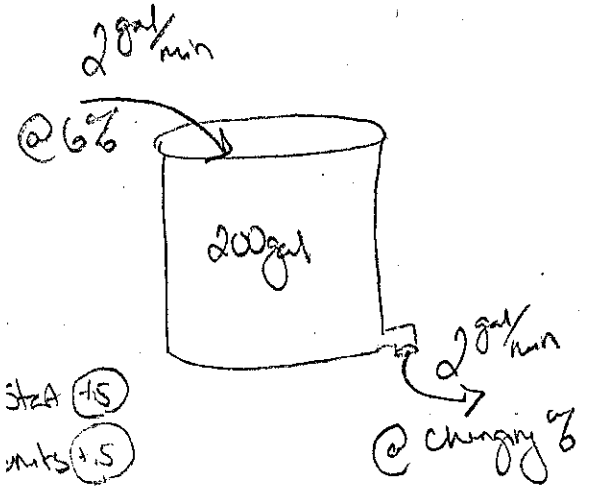
So

$u^2 = t^2 + \tan(t) + 25$

CK: $\frac{d}{dt}(u^2) = \frac{d}{dt}(t^2 + \tan(t) + c)$

$2u = 2t + \sec^2 t$ ✓

3. [4] (WebHW13 #7) A vat with 200 gallons of beer contains 4% alcohol (by volume). Beer with 6% alcohol is pumped into the vat at a rate of 2 gal/min and the mixture is pumped out at the same rate. Let $A(t)$ be the amount of alcohol in the vat at time t and set up a differential equation modeling the described situation. Do *not* solve the differential equation, but do justify the differential equation you set up.



start (1.5) units (1.5)

originally 4% alcohol
$A(t)$ is the amount of alcohol at time t
} initial condition
$A(0) = .04 \cdot 200$

Change in alcohol = alcohol coming in − alcohol going out (+1)

(1.5) alcohol coming in : $.06 \cdot 2 = .12$ $\left(\frac{\text{gal of alcohol}}{\text{min}}\right)$

(1.1) alcohol going out : percentage of alcohol in mix · rate

$\frac{A(t)}{200} \cdot 2 = \frac{A}{100}$ $\left(\frac{\text{gal of alcohol}}{\text{gal of mix}}\right)\left(\frac{\text{gal of mix}}{\text{min}}\right)\left(\frac{\text{gal of alcohol}}{\text{min}}\right)$

So $\underbrace{\frac{dA}{dt}}_{(1.5)} = .12 - \frac{A}{100}$

note: this is pretty nice to finish up

$\frac{A}{100}\,dA = .12\,dt$

$\Rightarrow \frac{1}{200}A^2 = .12t + C$

= use initial condition to finish up.