TMATH 125 Quiz 3

Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [3] (ip wks #2) Find $\int_0^1 5^t \sin(5^t)\,dt$.

sub +1.5 try +.5 new +.5 endpts +.5

$u = 5^t$

$du = 5^t (\ln 5)\,dt \Rightarrow \frac{1}{\ln 5}\,du = 5^t\,dt$

$$\int_0^1 5^t \sin(5^t)\,dt = \int_0^1 \sin(5^t)\,5^t\,dt = \int_{5^0}^{5^1} \sin(u)\,\frac{1}{\ln 5}\,du$$

$$= \int_1^5 \frac{1}{\ln 5}\sin u\,du = \frac{1}{\ln 5}\int_1^5 \sin u\,du = \frac{1}{\ln 5}(-\cos u)\Big|_1^5$$

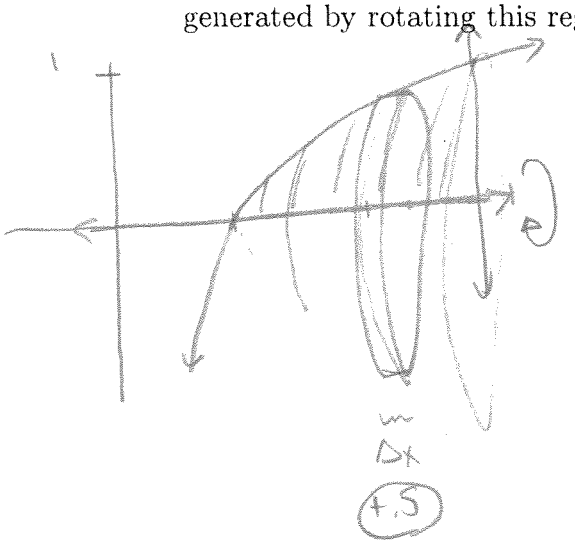
antider +.5

FTC II +.5

$$= \frac{1}{\ln 5}\left[-\cos 5 - (-\cos 1)\right] = \frac{1}{\ln 5}\left[\cos 1 - \cos 5\right]$$ alg/notation +.5

try integration knowledge correctly +.5

2. [3] (§7.1 #64) Consider the region bounded by the curves $y = \ln(x)$, $y = 0$, and $x = e$. Set up the integral (but do *not* integrate) that is used to find the volume of the solid generated by rotating this region about the x-axis.



+.5

$\pi r^2 \Delta x$

$\pi (y\text{ coord})^2 \Delta x$ } +1

$\pi (\ln x)^2 \Delta x$

Sum of approx cylinders

$$\int_1^e \pi (\ln x)^2\,dx$$ integral/notation +.5

+.5

3. [2] (WebHW9 #6) Find $\int 4\tan(x)\sec^3(x)\,dx$.

$= 4\int \tan(x)\sec^3(x)\,dx$

$= 4\int \sec^2(x)\cdot\tan(x)\sec(x)\,dx$

used right: (1.5) $\begin{cases} u = \sec x \\ du = \tan x \sec x\,dx \end{cases}$ picked right +.5

$= 4\int u^2\,du$

$= 4\cdot\frac{1}{3}u^3 = \frac{4}{3}\sec^3 x + C$ got +1.5

ck: $\left[\frac{4}{3}\sec^3 x + C\right]'$

$= \frac{4}{3}\cdot 3\sec^2 x\cdot\sec x\tan x + 0$

$= 4\sec^3(x)\tan x$ ✓

$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$

$= \frac{\cos x\cos x - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{1}{\cos^2 x} = \sec^2 x$

$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$

$= \frac{-1}{\cos^2 x}\cdot -\sin x = \frac{\sin x}{\cos^2 x}$

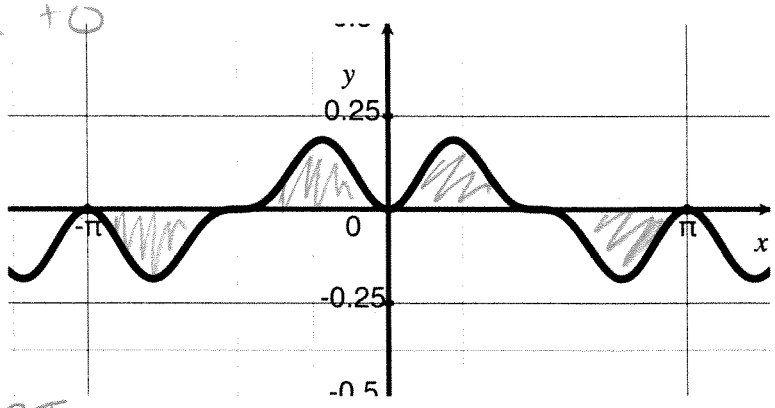
$= \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}$

$= \tan x\sec x$

$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\tan^2 x + 1 = \sec^2 x$

4. Consider $f(x) = \sin^2(x)\cos^3(x)$ shown on the right.



(a) [1] (§7.2 #55) Set up the integral (but do *not* to find the average value of the function on the interval $[-\pi, \pi]$.

average of y values

$\frac{1}{\pi - -\pi}\int_{-\pi}^{\pi} \sin^2(x)\cos^3 x\,dx$

(+.5) (+.5)

(b) [1] Use the graph and the definition of average value to find the average value of f on the interval $[-\pi, \pi]$.

0 there is an equal amount of shaded area above the x-axis as there is below