TMATH 125 Quiz 1

Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. Let $f(x) = 2 - \sqrt{4 - x^2}$.

   (a) [2] Draw the graph of f.

alg (.5) circle (1.5)

$y = 2 - \sqrt{4-x^2}$

$(y-2)^2 = (-\sqrt{4-x^2})^2$

$(y-2)^2 = 4 - x^2$

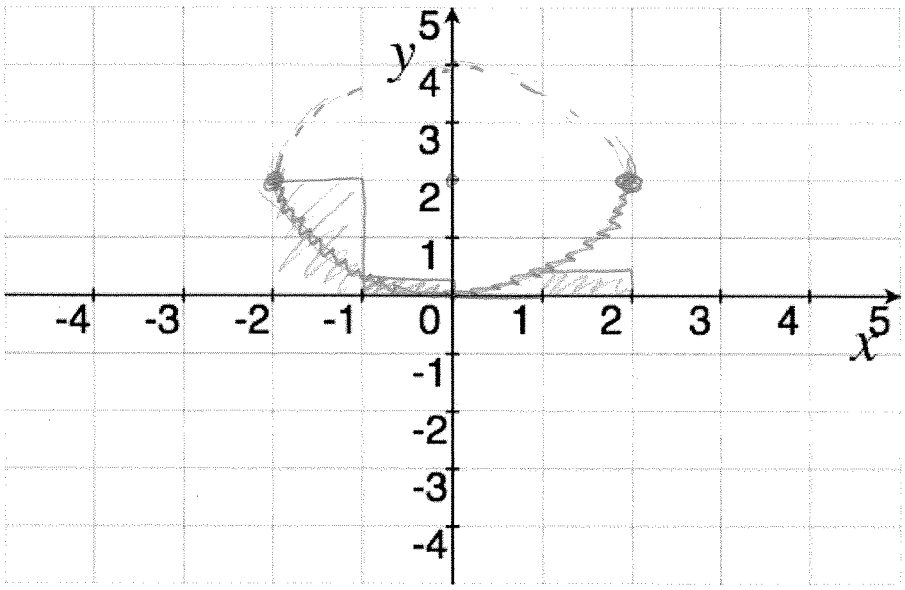
$+x^2 \quad +x^2$

$x^2 + (y-2)^2 = 4$

circle radius $\sqrt{4} = 2$ centred @ $(0,2)$ (+.5)

only the bottom of the circle

get it (+.5)



   (b) [2] (§5.1#2) Use your graph to estimate the area under the graph of f from -2 to 2 using four rectangles. Indicate if you are using right endpoints, left endpoints, or midpoints in your estimation.

Left hand approx (1.5)

$1 \cdot 2 + 1 \cdot (2-\sqrt{3}) + 1 \cdot 0 + 1 \cdot (2-\sqrt{3})$

$\approx 2 + .4 + 0 + .4$ (+1 approx pict)

$= 2.8$ (+.5 consistent)

pictured by shaded rectangles above

Right hand approx

$1 \cdot (2-\sqrt{3}) + 1 \cdot 0 + 1 \cdot (2-\sqrt{3}) + 1 \cdot 2$

$\approx .4 + 0 + .4 + 2$

$= 2.8$

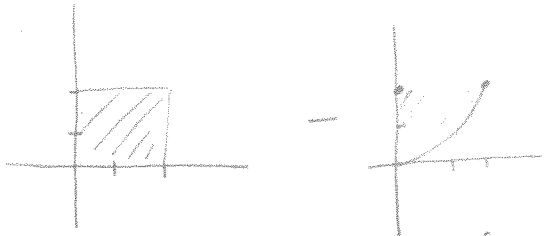
Midpoint Approx

$\approx 1.1 + 1.25 + 1.25 + 1$

$= 2.50$

   (c) [2] (WebHW2#3) Find $\int_0^2 f(x)\,dx$

(+1) Knew area looking at (+.5) area of quarter circle (+.5) alg



$= \int_0^2 f(x)\,dx$

square with area $2 \cdot 2$ − quarter of a circle with area $\frac{1}{4}\pi \cdot 2^2$ $= 4 - \pi$

2. [2] (FTC Wks) Find $\frac{d}{dx}\left(\int_1^{1+3x} \frac{t^3}{1+t^2}\,dt\right)$.

$g(x)$ = inside function: $1+3x$ (inside function)' : 3

$f(u)$ = outside function: $\int_1^u \frac{t^3}{1+t^2}\,dt$ (outside function)' = $\frac{u^3}{1+u^2}$ by FTC I (+.5)

By the chain Rule (+1)

$$\frac{d}{dx}\left(\int_1^{1+3x} \frac{t^3}{1+t^2}\,dt\right) = f'(g(x))g'(x) = f'(1+3x)\cdot 3$$

$$= \frac{(1+3x)^3}{1+(1+3x)^2} \cdot 3$$

got it (+.5)

3. [2] (WebHW3 #14) Evaluate $\int_0^{\frac{\pi}{4}} \sec(\theta)\tan(\theta)\,d\theta$.

Recall $\frac{d}{d\theta}(\sec\theta) = \frac{d}{d\theta}\left(\frac{1}{\cos\theta}\right) = \frac{d}{d\theta}\left[(\cos\theta)^{-1}\right]$

$= -(\cos\theta)^{-2}\cdot -\sin\theta = \frac{\sin\theta}{\cos^2\theta} = \tan\theta\cdot\sec\theta$

(+.5) So $\sec\theta$ is an antiderivative of $\sec\theta\tan\theta$

$$\int_0^{\pi/4} \sec(\theta)\tan(\theta)\,d\theta = \sec\theta\Big]_0^{\pi/4} = \sec(\pi/4) - \sec(0)$$

FTC II (+.5)

$$= \frac{1}{\cos\pi/4} - \frac{1}{\cos 0}$$

$$= \frac{1}{\frac{1}{\sqrt{2}}} - \frac{1}{1} = \sqrt{2} - 1$$

alg (+.5) notation (+.5)