Exam 2 **TMath 125** Spring 2012

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be continuous functions and x be a positive number.

   T F $\sqrt{x^2+4} = x+2$

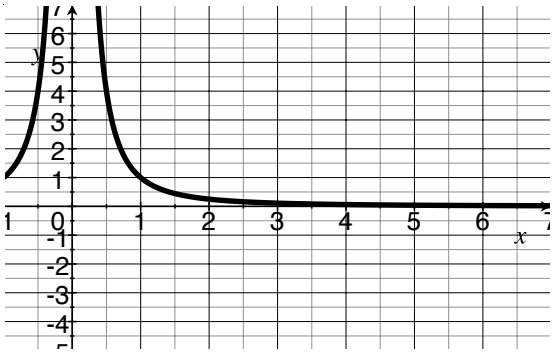
   T F $\displaystyle\int \frac{\cos x}{e^x}\,dx = \frac{\sin(x)}{e^x} + c$

   T F The area under a force function graphed with respect to distance, is the total work.

   T F All polynomials (with real coefficients) can be factored into linear and quadratic terms.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. The graph of $y = \frac{1}{x^2}$ is graphed below:



   (a) [2] Find the area trapped between $y = \dfrac{1}{x^2}$, the x-axis, from $x = 1$ to $x = 100$.

   (b) [1] Find $\displaystyle\lim_{b\to\infty} \int_1^b \frac{1}{x^2}\,dx$ and interpret your answer in terms of area.

3. For each of the following outline the method(s) you would use to find the general antiderivative. Include in the descriptions which substitutions you would make and the fall out that would occur. Essentially, give the same level of detail as was given on the practice exam key.
For extra credit, find the general antiderivative. (Each correct and complete answer will earn 1%.)

[3] (sorry-no meta-data for these)
$$\int \frac{t^5}{\sqrt{t^2+2}}\,dt$$

[3] (sorry-no meta-data for these)
$$\int \frac{x^3}{x^2+4x+3}\,dx$$

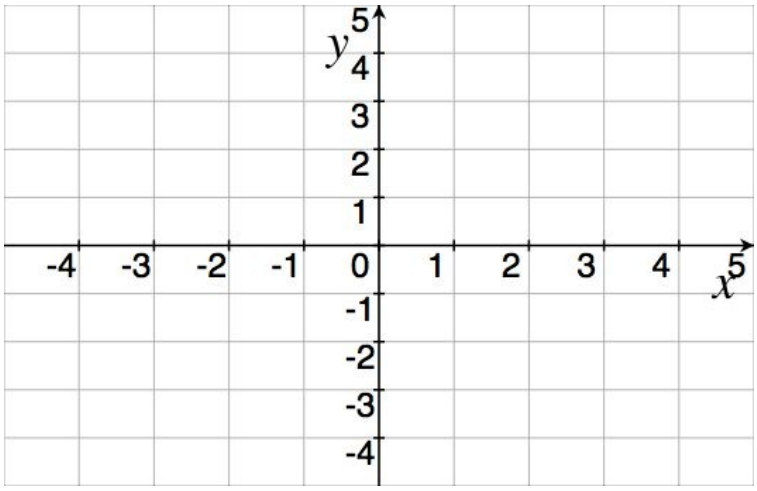
[3] (sorry-no meta-data for these)
$$\int \tan(\theta)\sec^3(\theta)\,d\theta$$

[3] (sorry-no meta-data for these)
$$\int v^3 e^{v^2}\,dv$$

4. (§7.1 #64) Consider the region bounded by the curves $y = 2\ln(x)$, $y = 0$, and $x = 3$.

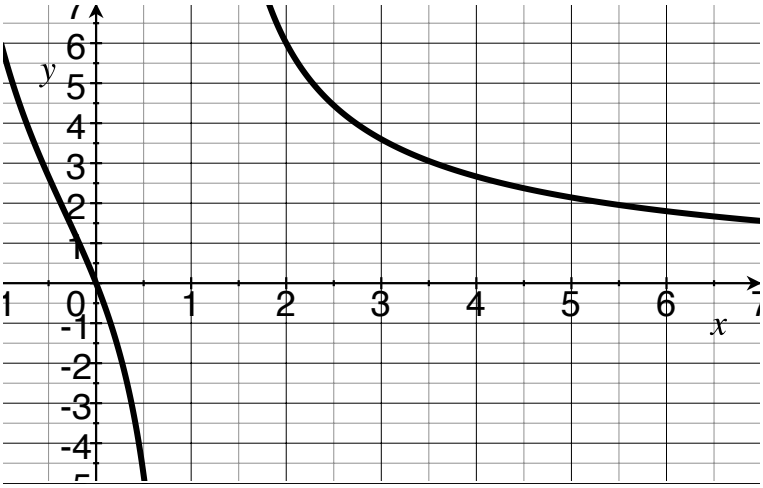
(a) [1] Carefully draw the region described above.

(b) [6] Find the volume that would result if the region was rotated about the line $x = 1$.



5. (word wks #4) Consider $g(x) = \dfrac{12x}{x^2 + x - 2}$ graphed below.

(a) [6] Find the average value of g on the interval $[2, 5]$. Call your answer f_{ave}.



(b) [2] Explain what $\int_2^5 g(x)\,dx$ and a 3 by f_{ave} rectangle have in common.

6. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
   No, doing both questions will not earn you extra credit.

   (a) (word wks #1) Recall that a circle is the collection of all (x, y) points a fixed distance r from a point (h, k). Use this and calculus to prove that the area of a circle with radius R is πR^2.

   (b) (word wks #5) A rocket accelerates by burning its onboard fuel, so the mass of the rocket decreases with time. Suppose the initial mass of the rocket at lift off (including its fuel) is m, the fuel is consumed at a rate r, and the exhaust gases are ejected with constant velocity v_c (relative to the rocket). A model for the velocity of the rocket at time t is given by the equation

$$v(t) = -gt - v_c ln\left(\frac{m - rt}{m}\right)$$

   where g is the acceleration due to gravity and t is not too large. If the rocket is on mars where gravity is 3.69 $\mathrm{m/s^2}$, $m = 30,000\mathrm{kg}$, $r = 160\frac{\mathrm{kg}}{\mathrm{s}}$, and $v_c = 3000\frac{\mathrm{m}}{\mathrm{s}}$, find the height of the rocket one minute after liftoff.