Key

Exam 1 — TMath 125 — Spring 2012

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be continuous functions and x be a positive number.

   T F $\frac{x^2-5}{x} = x-5$ $\quad x-5 = \frac{x-5}{1} = \frac{x(x-5)}{x\cdot 1} = \frac{x^2-5x}{x}$

   T F Approximating areas under curves with rectangles was first discovered in the 1990's by a biologist. rediscovered

   T F $\int \sin(x)\,dx = \cos(x) + c$ $\quad \frac{d}{dx}(\cos x + c) = -\sin x$

   T F $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

   T F $\int f(x)g(x)dx = \int f(x)\,dx \int g(x)dx$ $\quad \int x\cdot x\,dx$ vs $\int x\,dx \cdot \int x\,dx$; $\frac{1}{3}x^3 \neq \frac{1}{2}x^2 \cdot \frac{1}{2}x^2 + c$

   T F $\int_a^b f(x)+g(x)\,dx = -\int_a^c f(x)+g(x)\,dx + \int_c^b f(x)+g(x)\,dx$
   "$\int_a^c f(x)+g(x)dx + \int_c^b f(x)+g(x)dx$"

   T F $\int_{-1}^1 \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_{-1}^1 = \frac{-1}{1} - \frac{-1}{-1} = -2$

   should be a positive area

   T F If v is a velocity function, $\int v(t)\,dt + v(0)$ is the position function. velocity at time zero (not position)
   change in position between $t=0$ + $t=x$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [5] You have a friend in Calculus 1 this term who has just learned what a derivative is. Explain the Fundamental Theorem of Calculus (either one) to them.

(+1) start with FTC of some kind
(+1.5) makes sense mathematically
(+1) grammar/English issue
(+1.5) use words/symbols appropriate to audience

3. Consider the function shown below and to the right:

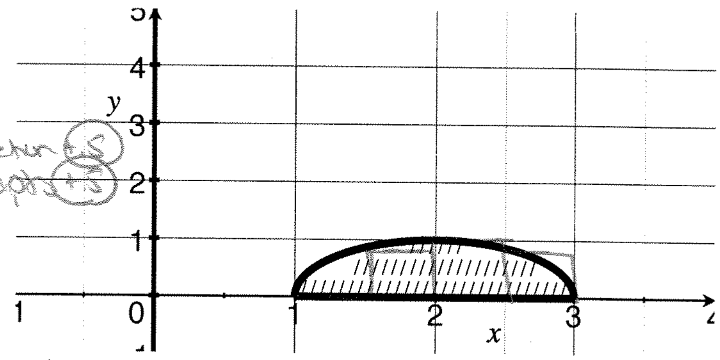
(a) [2] (def. int. wks #2) Describe the shaded area as a definite integral. The shaded region is a semicircle.

$\int^{3} \sqrt{1-(x-2)^2}\, dx$ notation (+.5) endpts (+.5)

Circle centered at (2,0) radius 1

$(x-2)^2+(y-0)^2=1$

$y=\sqrt{1-(x-2)^2}$ (+1)



(b) [3] (WebHW2 #6) Use four rectangles with a base length of .5 to approximate the area. Specify if you are using left-hand, right-hand, or midpoint approximations

left hand (+.5) consistent (+.5)

$0 \cdot .5 + .8 \cdot .5 + 1 \cdot .5 + .8 \cdot .5 =$

right-hand

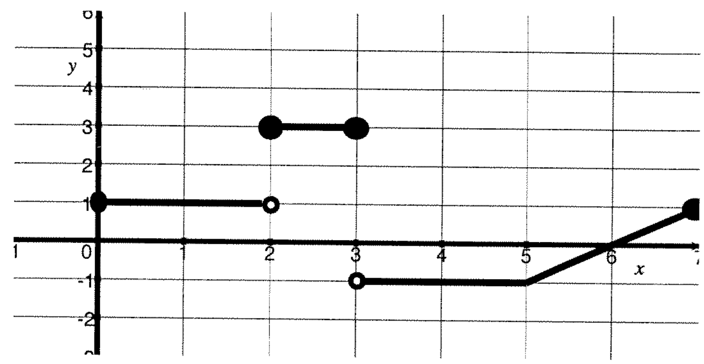
$.8 \cdot .5 + 1 \cdot .5 + .8 \cdot .5 + 0 \cdot .5$

width (+1)

height (+1)

4. Consider the piecewise-defined function f defined below

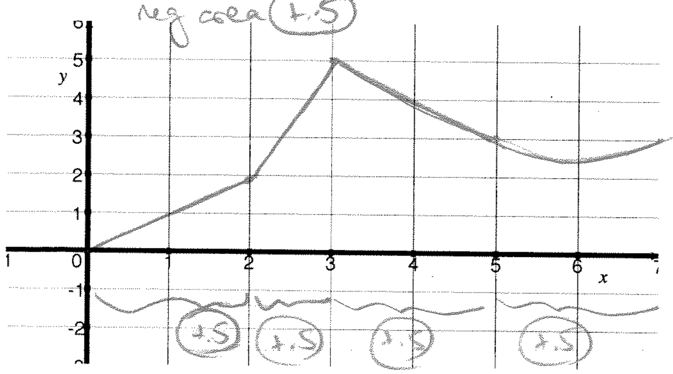
$$f(x) = \begin{cases} 1 & \text{if } 1 \le x < 2 \\ 3 & \text{if } 2 \le x \le 3 \\ -1 & \text{if } 3 < x \le 5 \\ x-6 & \text{if } 5 < x \le 7 \end{cases}$$



a) [3] Sketch the graph of $F(x) = \int_0^x f(x)\, dx$.

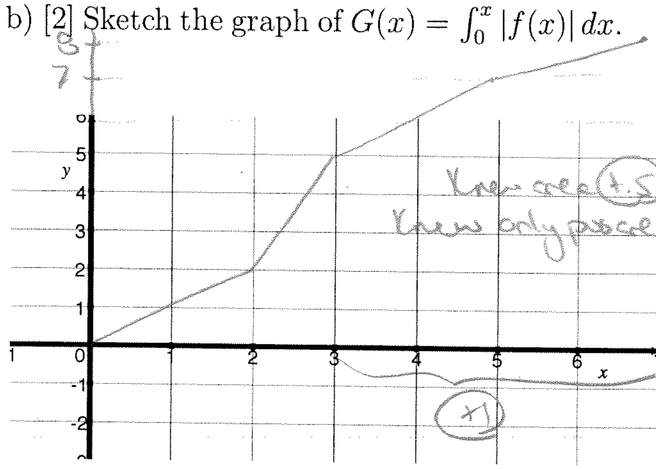
Knew area (+.5)

neg area (+.5)



(+.5) (+.5) (+.5) (+.5)

b) [2] Sketch the graph of $G(x) = \int_0^x |f(x)|\, dx$.



Knew area (+.5)

Knew only pos area (+.5)

(+1)

5. [2] (WebHW5 #10) If g is continuous and $\int_0^{14} g(x)\, dx = 4$, find $\int_0^7 f(2x)\, dx$.

Typo →

$u = 2x$

$du = 2dx$ (+1)

$\frac{1}{2} du = dx$

$\int_0^7 f(2x)\, dx \Rightarrow \int_{2 \cdot 0}^{2 \cdot 7} f(u) \frac{1}{2} du = \frac{1}{2} \int_0^{14} f(u)\, du = \frac{1}{2} \cdot 4 = 2$

stated sub (+.5)

notation (+.5)

6. Find the limit or explain why it does not exist.

[2] (Quiz2 #2)

$$\int \frac{\sin(\ln(13x))}{x}\,dx$$

$u = \ln(13x)$

$du = \frac{1}{13x} \cdot 13 \cdot dx \Rightarrow du = \frac{1}{x} dx$ } sub (+.5) right (+.5)

$\int \frac{\sin(\ln(13x))}{x} dx = \int \sin(\ln(13x)) \frac{1}{x} dx$

$= \underbrace{\int \sin(u)\,du}_{\text{sub (+.5)}} = -\cos u + c$ antider (+.5)

$= -\cos(\ln(13x)) + c$

[3] (WebHW5 #9)

$$\int_{e^{64}}^{e^{81}} \frac{dx}{x\sqrt{\ln(x)}}$$

sub (+.5)
right (+.5)
limits (+.5)
notation (+.5)

$u = \ln x$

$du = \frac{1}{x} dx$

$\int_{e^{64}}^{e^{81}} \frac{1}{\sqrt{\ln x}} \cdot \frac{1}{x} dx = \int_{\ln(e^{64})}^{\ln(e^{81})} \frac{1}{\sqrt{u}} du$

$= \int_{64}^{81} u^{-1/2} du = 2u^{1/2}\Big]_{64}^{81}$ } integral (+.5)

$= 2(\sqrt{81} - \sqrt{64}) = 2(9-8) = 2$

FTC2 (+.5)

[3] (§5.3 #16)

$$\frac{d}{dx}\left(\int_0^{x^4} \cos(\theta)\,d\theta\right)$$

use chain (+.5)

Chain Rule

.5 set up { $g(x) = x^4$, $f(u) = \int_0^u \cos\theta\,d\theta$ }

$g'(x) = 4x^3$ } (+.5)

$f'(u) = \cos u$ } (+.5)
$\hookrightarrow$ by FTC I

notation (+.5)

$f'(g(x))g'(x) = f'(x^4) \cdot 4x^3$

$= \cos(x^4) \cdot 4x^3$

$= \underbrace{4x^3 \cos(x^4)}_{\text{got it (+.5)}}$

[2] (WebHW4 #6)

$$\int 3v(v^2+6)\,dv$$

$\int 3v^3 + 18v\,dv$

$\underbrace{\frac{3}{4}v^4}_{(+.5)} + \underbrace{9}_{(+.5)}v^2 + \underbrace{c}_{(+.5)}$

dist alg (+.5)

zero's 0 & 4 $\quad$ zero's 0 & 2

$y(y-4)=x$ $\quad$ $y(2-y)=x$

7. (Area Wks) Consider the region bounded by the cuves $y^2 - 4y = x$ and $2y - y^2 = x$.

$y^2-4y=2y-y^2$

$2y^2-6y=0$

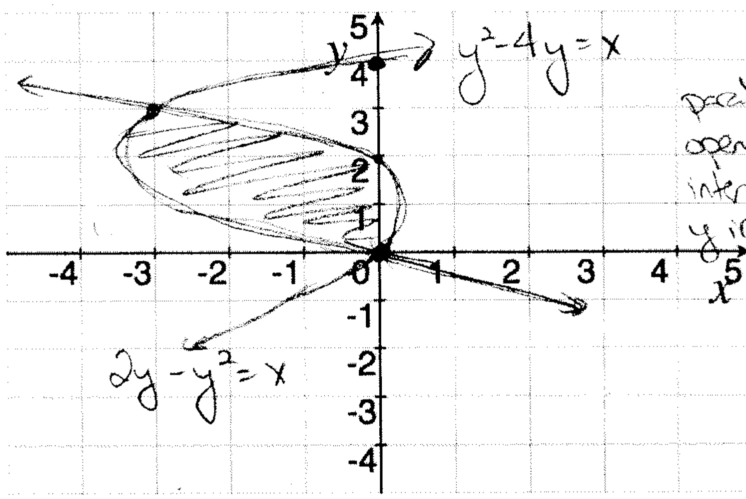
$y(y-3)=0$

$y=0$ & 3

$x=0$ & -3

(a) [2] Carefully draw the region described above.

(b) [2] Set up the integral *but do not find/integrate* that corresponds to the area sketched in part (a).



parabolas (.5)
opening (.5)
intersections (.5)
y intercepts (.5)

$$\int_0^3 (2y-y^2)-(y^2-4y)\,dy$$

minus (.5)
order (.5)
endpoints (.5)

notation (.5)

$\to$ sec(x) graph shifted up 1 unit.

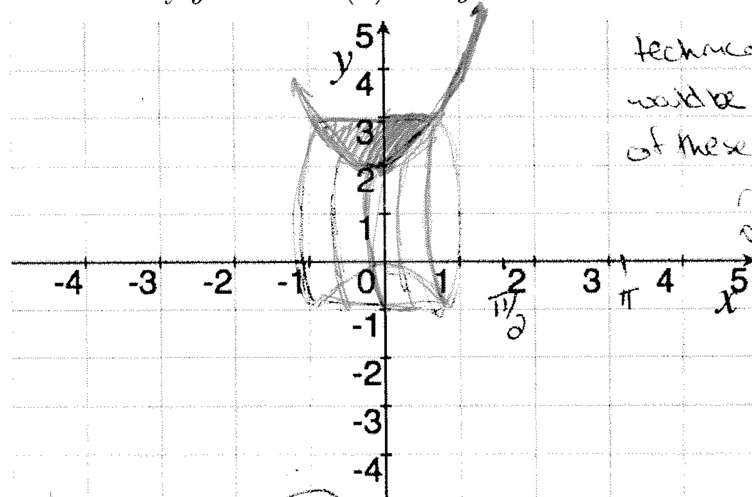
8. (§6.2 #13) Consider the region bounded by $y = 1 + \sec(x)$ and $y = 3$.

x	1+sec x
0	$1+\frac{1}{\cos 0}=2$
$\pi/4$	$1+\frac{1}{\cos \pi/4} = 1+\sqrt{2}$
$\pi/2$	vert asy

$(\pi/3, 3)$

(a) [1] Sketch the region described above.

(b) [4] Consider the above region rotated about the line $y = 1$. Use calculus to find the volume of the resulting solid.



technically there would be an infinite # of these tube things

Outside Volume − Inside volume (.5)

Cylinder with radius 2 & height $2\pi/3$ − Sum of approx cyl $\pi r_I^2 \Delta x$

Sum of approx cyl

$\underbrace{\pi \cdot 2^2 \cdot \frac{2\pi}{3}} - \pi\left(1+\sec(x)-1\right)^2 \Delta x$

$8\frac{\pi^2}{3}$ (.5) $- 2\underbrace{\int_0^{\pi/3} \pi(\sec x)^2\,dx}$ (1.5)

$3 = 1+\sec x$

$2 = \sec x$

$2 = \frac{1}{\cos x}$ (.5)

$\frac{1}{2} = \cos x$

$\to x = \frac{\pi}{3}, -\frac{\pi}{3}$

$\tan \pi/3 = \frac{\sqrt{3}/2}{1/2}$

$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$

$= \sin x \frac{-1}{\cos x} \cdot -\sin x + \frac{\cos x}{\cos x}$

$= \tan^2 x + 1$

$\frac{\sin^2 x + \cos^2 x}{\cos^2} = \frac{1}{\cos^2}$

$\tan^2 x + 1 = \sec^2 x$

$= \frac{8}{3}\pi^2 - 2\pi \tan x \Big]_0^{\pi/3}$ $\quad$ antider. (.5)

FTC2 (.5)

$= \frac{8}{3}\pi^2 - 2\pi\left(\tan \frac{\pi}{3} - \tan 0\right) = \frac{8}{3}\pi^2 - 2\pi\left(\sqrt{3}\right)$

9. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (Story Wks #5) A 230 foot cable that weighs .48 pounds per foot and hangs vertically from the end of a crane. (Such a cable has a capacity of 23,000 lbs!)

i. [3] How much work does it it take to draw the cable towards the top of the crane to that only 150 feet is hanging vertically from the crane?

ii. [3] Workers attach a 10,000lb concrete tube to the bottom of the (extended) cable. (So the crane is 230 feet above the end of the cable/to of the tube.) How much work would it take to draw the cable (and thus the concrete tube) up ~~100~~ 80 feet.

230
−150
80

80
150

(b) While sleeping, breathing is roughly cyclic. The air flowing in and our of the lungs is well modeled by the function $\frac{3}{5}\sin\left(\frac{2\pi t}{7}\right)$ liters per second.

i. [1] How long does it take to complete one breath cycle? (Breath in and out).

ii. [2] (Story wks #6) Given that at time 0 there is 30 ml, find the volume of air in the lungs at time t.

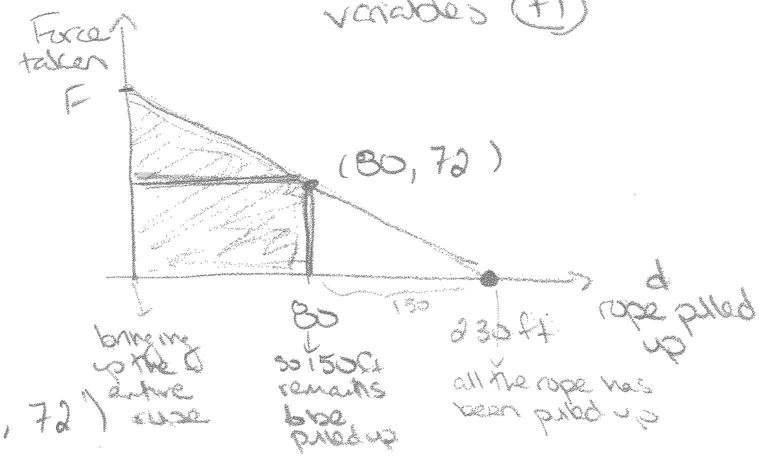
iii. [3] (Story wks 7) Find the average volume of air inhaled during the first half of the breathing cycle.

variables (+1)

a) i) Work = Force · dist x .5

230ft · .48 lb/ft = 110.4 lbs

corresponds to the area under the force function shown to the right



function (+1) $\left\{ F = \frac{-110.4}{230} d + 110.4 = -.48d + 110.4 \right.$

endpts (.5) so $(80, -.48\cdot 80 + 110.4) = (80, 72)$ is on the graph

graph (.5) => Work is $80\cdot 72 + \frac{1}{2}\cdot 80\cdot(110.4-72) = 5760 + 1536 = 7296$

ii) If we bring the cable up 80 ft that will leave 150ft hanging from the crane => we need only add the work it takes to lift the concrete tube 80ft to our answer in i) (+1)

work to lift tube + work to lift cable

$\underbrace{80\text{ft}\cdot 10{,}000\text{ lb}}_{(+1)} + \underbrace{7296}_{\text{answer from i}} = 807296 \text{ ft·lbs}$

(+.5)

b) i) ie when does $\frac{3}{5}\sin\left(\frac{2\pi t}{7}\right)$ complete a cycle?

(+.5) normal period of sin: 2π

so when does $\frac{2\pi t}{7} = 2\pi \Rightarrow t = 7$

(+.5) 7 seconds

ii) (Vol of air in lungs)$' = \frac{3}{5}\sin\left(\frac{2\pi t}{7}\right)$ (+.5)

So $\int \frac{3}{5}\sin\left(\frac{2\pi t}{7}\right)dt$ $\qquad u = \frac{2\pi}{7}t$

$du = \frac{2\pi}{7}dt \Rightarrow \frac{7}{2\pi}du = dt$

(+1) $\Rightarrow \frac{3}{5}\int \sin u \frac{7}{2\pi}du$

$\Rightarrow \frac{21}{10\pi}\int \sin u\,du = \frac{-21}{10\pi}\cos u + c = \frac{-21}{10\pi}\cos\left(\frac{2\pi t}{7}\right) + c$

Since Vol at $t=0$ is 30mL or $30\cancel{mL}\cdot\frac{1L}{1000\cancel{mL}} = .03L$

(+.5) $\frac{-21}{10\pi}\cos\left(\frac{2\pi(0)}{7}\right) + c = .03L$

$\Rightarrow c = .03 + \frac{21}{10\pi} \approx$

So Volume at time t is: $\frac{-21}{10\pi}\cos\left(\frac{2\pi t}{7}\right) + \left(.03 + \frac{21}{10\pi}\right)$

iii) $\underbrace{\frac{1}{3.5-0}}_{(+.5)}\underbrace{\int_0^{3.5}\frac{3}{5}\sin\left(\frac{2\pi t}{7}\right)dt}_{(+.5)} = \frac{1}{3.5}\underbrace{\frac{-21}{10\pi}\cos\left(\frac{2\pi t}{7}\right)\Big]_0^{3.5}}_{\text{from above } (+.5)}$

computed 3.5 (+.5)

$= \frac{-21}{35\pi}\left[\cos\left(\frac{2\pi\cdot 3.5}{7}\right) - \cos(0)\right] =$

FTC1 (+.5) $\qquad$ evaluate (+.5)