Exam 1

TMath 125

Spring 2012

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be continuous functions and x be a positive number.

$$T \quad F \quad \frac{x^2 - 5}{x} = x - 5$$

T F Approximating areas under curves with rectangles was first discovered in the 1990's by a biologist.

T F
$$\int \sin(x) dx = \cos(x) + c$$

T F
$$\int_{a}^{b} f(x) dx = -\int_{b}^{a} f(x) dx$$

T F $\int f(x)g(x)dx = \int f(x) dx \int g(x)dx$

T F
$$\int_{a}^{b} f(x) + g(x) dx = -\int_{a}^{c} f(x) + g(x) dx + \int_{c}^{b} f(x) + g(x) dx$$

T F
$$\int_{-1}^{1} \frac{1}{x^2} dx = \frac{-1}{x} \Big|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -2$$

T F If v is a velocity function, $\int v(t) dt + v(0)$ is the position function.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [5] You have a friend in Calculus 1 this term who has just learned what a derivative is. Explain the Fundamental Theorem of Calculus (either one) to them.

3. Consider the function shown below and to the right:



Specify if you are using left-hand, right-hand, or midpoint approximations

4. Consider the piecewise-defined function f defined below

$$f(x) = \begin{cases} 1 & \text{if } 1 \le x < 2\\ 3 & \text{if } 2 \le x \le 3\\ -1 & \text{if } 3 < x \le 5\\ x - 6 & \text{if } 5 < x \le 7 \end{cases}$$



a) [3] Sketch the graph of $F(x) = \int_0^x f(x) \, dx$.

b) [2] Sketch the graph of $G(x) = \int_0^x |f(x)| dx$.



6. Find the limit or explain why it does not exist.

[2] (Quiz2 #2)

$$\int \frac{\sin(\ln(13x))}{x} dx$$
[3] (WebHW5 #9)

$$\int_{e^{64}}^{e^{81}} \frac{dx}{x\sqrt{\ln(x)}}$$

$$\begin{bmatrix} 3 \end{bmatrix} (\$5.3 \ \#16) \\ \frac{d}{dx} \left(\int_0^{x^4} \cos(\theta) \, d\theta \right)$$

[2] (WebHW4 #6) $\int 3v(v^2 + 6) \, dv$

7. (Area Wks) Consider the region bounded by the cuves $y^2 - 4y = x$ and $2y - y^2 = x$.

- (a) [2] Carefully draw the region described above.
- (b) [2] Set up the integral but do not find/integrate that corresponds to the area sketched in part (a).

				v^{5}					
				- 4 3					
			-	2					
				1			2		575
-4	-3	-2	-1	0	1	2	3	4	5
				-1					$-\lambda$
				-2					
				-3			14		
				-4					

- 8. (§6.2 #13) Consider the region bounded by $y = 1 + \sec(x)$ and y = 3.
 - (a) [1] Sketch the region described above.
 - (b) [4] Consider the above region rotated about the line y = 1. Use calculus to find the volume of the resulting solid.

				v^{5}					
				· 4					
				3					
				2					
				1					
-4	-3	-2	-1	0	1	2	3	4	1
				-1					-1
				-2					
				-3					
				-4					

- [6] Choose ONE of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit. No, doing both questions will not earn you extra credit.
 - (a) (Story Wks #5) A 230 foot cable that weighs .48 pounds per foot and hangs vertically from the end of a crane. (Such a cable has a capacity of 23,000 lbs!)
 - i. [3] How much work does it it take to draw the cable towards the top of the crane to that only 150 feet is hanging vertically from the crane?
 - ii. [3] Workers attach a 10,000lb concrete tube to the bottom of the (extended) cable. (So the crane is 230 feet above the end of the cable/to of the tube.) How much work would it take to draw the cable (and thus the concrete tube) up 100 feet.
 - (b) While sleeping, breathing is roughly cyclic. The air flowing in and our of the lungs is well modeled by the function $\frac{3}{5}\sin\left(\frac{2\pi t}{7}\right)$ liters per second.
 - i. [1] How long does it take to complete one breath cycle? (Breath in and out).
 - ii. [2] (Story wks #6) Given that at time 0 there is 30 ml, find the volume of air in the lungs at time t.
 - iii. [3] (Story wks 7) Find the average volume of air inhaled during the first half of the breathing cycle.