$$\int \cot(x)\csc^2(x)\,dx$$

$u = \cot(x)$

$du = (\cot(x))' = \left(\frac{\cos x}{\sin x}\right)'$

$= \frac{-\sin x \sin x - \cos(x)\cos(x)}{\sin^2 x}\,dx$

$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}\,dx$

$= \frac{-1}{\sin^2 x}\,dx = -\csc^2 x\,dx$

so

$$\int \cot(x)\csc^2(x)\,dx = \int u(-1)\,du$$

$= -\frac{1}{2}u^2 + C$

$= -\frac{1}{2}\cot^2(x) + C$

Ck: $\left[-\frac{1}{2}\cot^2(x) + C\right]' = \frac{-1}{\cancel{2}}\cdot\cancel{2}\cot(x)\csc(x)$ ✓

$$\int \cot^2(x)\csc^4(x)\,dx$$

$u = \cot(x)$

$du = -\csc^2 x\,dx$

$-du = \csc^2 x\,dx$

$= \int u^2 \csc^2(x)\csc^2(x)\,dx$

$= \int u^2 \underbrace{\csc^2(x)}\,(-du)$

$= -\int u^2 (1 + \cot^2(x))\,du$

recall

$\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\underbrace{1 + \cot^2 x = \csc^2 x}$

$= -\int u^2(1 + u^2)\,du$

$= -\int u^2 + u^4\,du$

$= -\frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$= -\frac{1}{3}\cot^3(x) - \frac{1}{5}\cot^5(x) + C$

Ck: $\left[-\frac{1}{3}\cot^3(x) - \frac{1}{5}\cot^5(x) + C\right]'$

$+\frac{1}{\cancel{3}}\cdot\cancel{3}\cot^2(x)\csc^2(x) + \frac{1}{\cancel{5}}\cdot\cancel{5}\cot^4(x)\csc^2(x) + 0$

$= \cot^2(x)\csc^2(x)\,(1 + \cot^2(x))$

$= \cot^2(x)\csc^2(x)\csc^2(x)$ ✓