Fix #1c, #5d

FINAL

Math 125

PRACTICE

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

1. Each of the following is wrong. Explain why.

   (a) $\int_a^b f(x)g(x)\,dx = \int_a^b f(x)\,dx * \int_a^b g(x)\,dx$

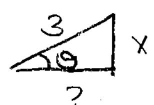
   Just like the product of derivatives is NOT the derivative of the products – so too with integrals! To undo products we make use of integration by parts: $\int u\,dv = uv - \int v\,du$

   (b) $\int_1^4 \frac{3+\sqrt{x}+x}{x}\,dx = \int_1^4 \frac{3}{x} + x^{\frac{-1}{2}} + 1\,dx = 3\ln(x) + 2x^{\frac{1}{2}} + x + c$

   We are given a definite integral so the end result should be a number not a family of functions! In particular, we need to use the FTC II to plug in values & perform a subtraction.

   (c) One problem required the substitution of $x = 3\csc(\theta)$. Then $\cos(\theta) = \frac{\sqrt{9-x^2}}{3}$.

   $x = 3\csc\theta$ Sohcahtoa $?^2 + x^2 = 3^2$ So

   $x = \frac{3}{\sin\theta}$ 

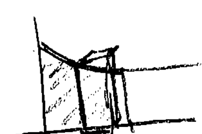
   $?^2 = 9 - x^2$ $\cos\theta = \frac{adj}{hyp} = \frac{\sqrt{9-x^2}}{3}$

   $\sin\theta = \frac{x}{3}$ $? = \sqrt{9-x^2}$

   Oh! this is correct!

   (d) The volume whose base is bounded by $y = 0$, $x = 0$, $x = 1$ and $f(x) = \frac{1}{x^2+3x+2}$ and has square cross sections perpendicular to the x-axis has volume equal to $\int_0^1 \pi \left(\frac{1}{x^2+3x+2}\right)^2 dx$.

   The above assumes the cross sections are circular but the cross sections are square! Thus the answer is: $\int_0^1 \left(\frac{1}{x^2+3x+2}\right)^2 dx$

2. Explain the second Fundamental Theorem of Calculus.

   We can find a definite integral $\int_a^b f(x)\,dx$ (area bounded by $f(x)$, the x-axis & vert. lines $x=a$ & $x=b$) provided f is continuous by finding an antiderivative $F(x)$ of $f(x)$ and computing $F(b) - F(a)$.

3. Find $\frac{d}{dx}\int_0^{x^2+3x} e^{t^2}\,dt$ Chain Rule? let $u = x^2+3x$

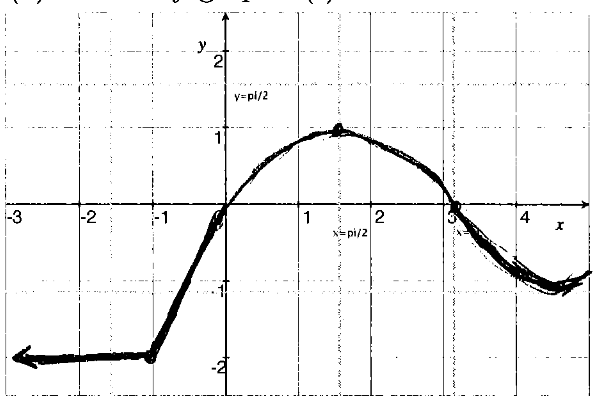
$$= \frac{d}{du}\int_0^{u} e^{t^2}\,dt \cdot \frac{du}{dt}$$

$$= e^{u^2}\cdot(2x+3) = e^{(x^2+3x)^2}(2x+3)$$

4. Let v be the function that records the velocity of a particle which is well approximated by the following formula.

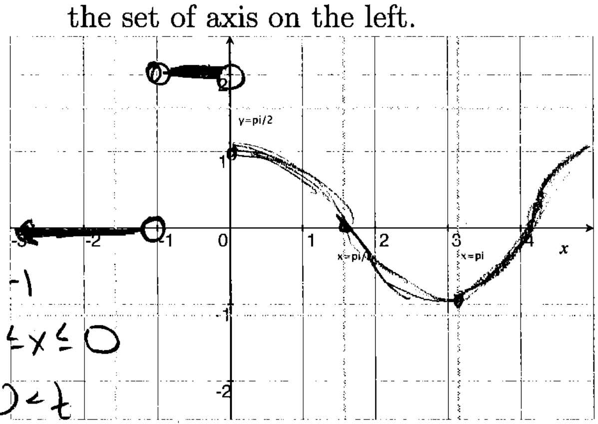
$$v(t) = \begin{cases} -2 & t \le -1 \\ 2t & \text{if } -1 \le x \le 0 \\ \sin t & \text{if } 0 < t \end{cases}$$

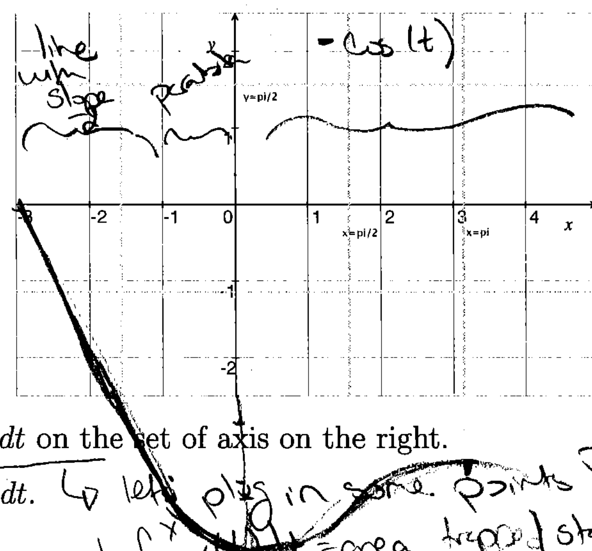
(a) Carefully graph $v(t)$ on the set of axis.



(b) Give a rough sketch of the function recording the acceleration of the particle on the set of axis on the left.

acceleration $= v'(t) = \frac{d}{dt}(v(t)) = \begin{cases} 0 & t \le -1 \\ 2 & -1 \le x \le 0 \\ \cos(t) & 0 < t \end{cases}$





(c) Give a rough sketch of the graph $\int_{-3}^{x} v(t)\,dt$ on the set of axis on the right.

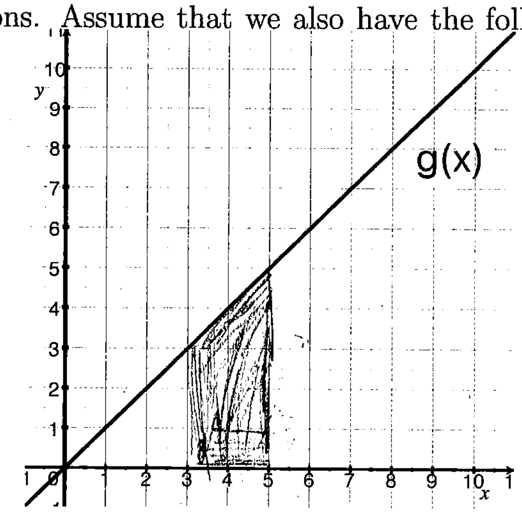
(d) Describe the physical meaning of $\int_{-3}^{x} v(t)\,dt$. ↳ left plug in some points?

The distance traveled from the initial position (where the initial position is at time -3)

x	$\int_{-3}^{x} v(t)\,dt$ = area trapped standing at
-3	$\int_{-3}^{-3} v(t)dt = 0$
-2	$\int_{-3}^{-2} v(t)dt$ = rectangle area $= 1\cdot 2 = 2$
-1	$\int_{-3}^{-1} v(t)dt$ = rect. area $= 2\cdot 2 = 4$
0	$\int_{-3}^{0} v(t)dt$ = rect + Δ $= 4 + \frac{1}{2}(2)(1) = 5$

5. Let g be the line graphed below on the right. Let f be a function that is continuous and twice differentiable to continuous functions. Assume that we also have the following values for f and f'.

x	$f(x)$	$f'(x)$
0	2	3
4	7	5



(a) Find $f(4)$.

7

(b) Find $g'(4)$.

rule $g(x) = x \Rightarrow g'(x) = 1$

So $g'(4) = 1$

(c) Evaluate $\int_3^5 g(x)\,dx$

$= \int_3^5 x\,dx = \frac{1}{2}x^2\Big]_3^5 = \frac{25}{2} - \frac{9}{2} = \frac{16}{2} = 8$

or. area shaded above $3 \cdot 2 + \frac{1}{2} \cdot 2 \cdot 2$

(d) Find the rule for $\frac{d}{dx}\left(\int_0^x g(t)\,dt\right)$ for $x > 0$.

(FTC I) $= g(x) = x$

(e) Evaluate $\int_0^4 f'(x) + 1\,dx$ $= \int_0^4 f'(x)\,dx + \int_0^4 1\,dx$

(FTC II) $= f(x)\Big]_0^4 + x\Big]_0^4 = f(4) - f(0) + [4 - 0]$

$= (7 - 2) + 4 = 5 + 4 = 9$

(f) Given $\int_0^2 f'(x)\,dx = 1$, find $\int_2^4 f'(x)\,dx$

$\int_0^4 f'(x)\,dx - \int_0^2 f'(x)\,dx = f(x)\Big]_0^4 - 1 = (7-2) - 1$

$= 4$

(g) Evaluate $\int_0^4 g(x) f''(x)\,dx$

integration by parts?

$u = g(x) = x$ $\quad v = f'(x)$

$du = dx$ $\quad dv = f''(x)\,dx$

$x f'(x)\Big]_0^4 - \int_0^4 f'(x)\,dx$

$[4f'(4) - 0f'(0)] - f(x)\Big]_0^4$

$[4 \cdot 5 - 0] - [f(4) - f(0)]$

$20 - (7 - 2) = 20 - 5 = 15$

6. For each of the following outline the method(s) you would use to find the general antiderivative. For example, if you think trigonometric substitution would work write "trigonometric substitution" and identify what substitution (u) you would use.

$\int_0^{\frac{\pi}{4}} \sec^4 x \tan^4 x \, dx = \int_0^{\pi/4} \sec^2 x \tan^4 x \sec^2 x \, dx$

reserve $\sec^2 x \, dx$ for du
and let $\tan(x) = u$

use pythagorus $\tan^2\theta + 1 = \sec^2\theta$ to turn the remaining $\sec^2 x$ factor into something involving $\tan^2 x$.

Change limits to $\int^{\tan \pi/4} (u^2+1) u^4 \, du$

and finish with FTC II

$\int x \cos^2 x \, dx$ integration by parts coupled w/ trig identities

$= \int x \underbrace{\tfrac{1}{2}(1+\cos 2x)}_{\text{trig identity - double angle ID}} dx$

$= \underbrace{\int \tfrac{1}{2} x \, dx}_{\text{use power rule}} + \int \tfrac{1}{2} x \cos(2x) \, dx$

$u = x$ $\quad v = \tfrac{1}{2}\sin(2x)$

$du = dx$ $\quad dv = \cos(2x)\, dx$

$\int_1^\infty \frac{1}{x^2} dx = \lim_{t \to \infty} \underbrace{\int_1^t \frac{1}{x^2} dx}_{\text{can use power rule}}$

use limits b/c improper integral?

$= \lim_{t\to\infty} \left.\frac{-1}{x}\right]_1^t$ then use FTC II

lastly evaluate the limit

$\int \frac{1}{x^2\sqrt{x^2+4}} dx$ trigonometric substitution

$x = 2\tan\theta$

$dx = 2\sec^2\theta \, d\theta$

use pythagorus and restrict θ so that $\sec\theta = \sqrt{\tan^2\theta + 1}$

We'll have something like $\int \frac{\sec^2\theta}{\tan^2\theta \sec\theta} d\theta$

We can simplify to just sin's & cos's to finish integration w/ substitution.
Make a $\triangle$ in order to return to x's.

$\int_0^3 \frac{1}{x-1} dx$ note $\frac{1}{x-1}$ is NOT continuous at 1?

We need to use an improper integral definitions and limits

$= \int_0^1 \frac{1}{x-1} dx + \int_1^3 \frac{1}{x-1} dx$

$= \lim_{b\to 1^-} \int_0^b \frac{1}{x-1} dx + \lim_{a \to 1^+} \int_a^3 \frac{1}{x-1} dx$

integrate definite integrals w/ $u = x-1$ and ln's.

lastly evaluate the limit.

$\int \frac{17x-1}{2x^2+3x-2} dx$ partial fractions

find A and B so that

$\frac{A}{(2x-1)} + \frac{B}{(x+2)} = \frac{17x-1}{(2x-1)(x+2)}$

Then use substitution & ln's to finish up?

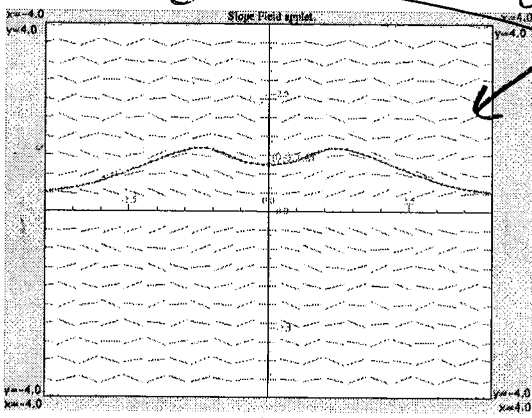
7. Match the differential equations with the solutions graphs. *Justify* your choice.

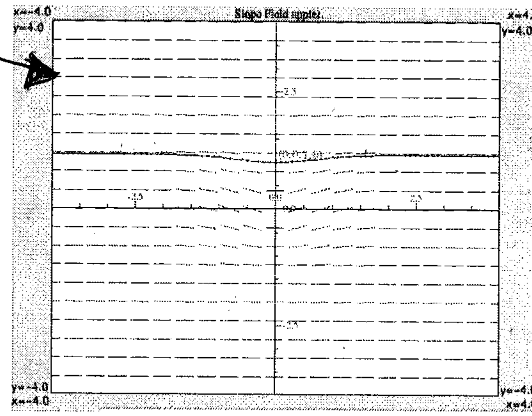
(a) $y' = xe^{-x^2-y^2}$ $= xe^{-(x^2+y^2)}$ $= \frac{x}{e^{x^2+y^2}}$

@ $(5,5) \Rightarrow$ slope $\frac{5}{\text{big}}$ positive but close to zero

(b) $y' = \sin(xy)\cos(xy)$

Note slope $= 0$ when $x \cdot y = 0, \pi, 2\pi, 3\pi \ldots$ and when $x \cdot y = \frac{\pi}{2}, \frac{3\pi}{2}$, etc





8. Write the following in sigma notation $-\frac{1}{3} + \frac{3}{7} - \frac{1}{2} + \frac{5}{9} - \frac{3}{5} + \frac{7}{11}$

$(-1)\frac{2}{6} + \frac{3}{7} + (-1)\frac{4}{8} + \frac{5}{9} + (-1)\frac{6}{10} + \frac{7}{10}$

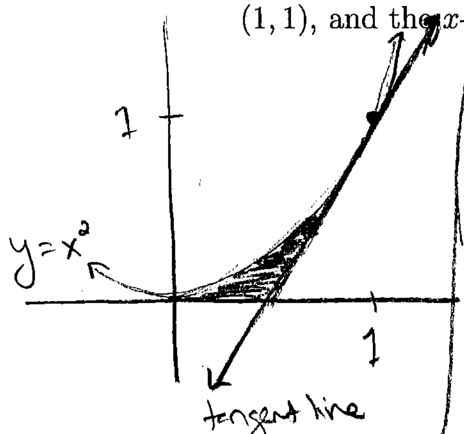
$$\sum_{i=1}^{6} (-1)^i \frac{(i+1)}{(i+5)}$$

9. Expand $\displaystyle\sum_{i=-3}^{1} \frac{i\ln(x)}{(-1)^i(5+i)}$

$i=-3$ $\quad i=-2$ $\quad i=-1$ $\quad i=0$ $\quad i=1$

$$\frac{-3\ln(x)}{(-1)(5-3)} + \frac{-2\ln(x)}{(5-2)} + \frac{-1\ln(x)}{(-1)(5-1)} + 0 + \frac{\ln(x)}{(-1)(5+1)}$$

10. Find the area of the region bounded by $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x-axis.



$y = 2x - 1$

or $\frac{1}{2}(y+1) = x$

finding the tangent line: Looking for $y = mx + b$

m = slope of line tang to f @ x=1 $= f'(1)$

$= 2x\Big|_{x=1} = 2$

thru $(1,1)$ so

$1 = 2(1) + b$

$\Rightarrow b = 1 - 2 = -1$

I'll integrate with respect to y

$\int_0^1$ tangent line $-$ x-coord dy

$= \int_0^1 \frac{1}{2}(y+1) - \sqrt{y}\, dy$

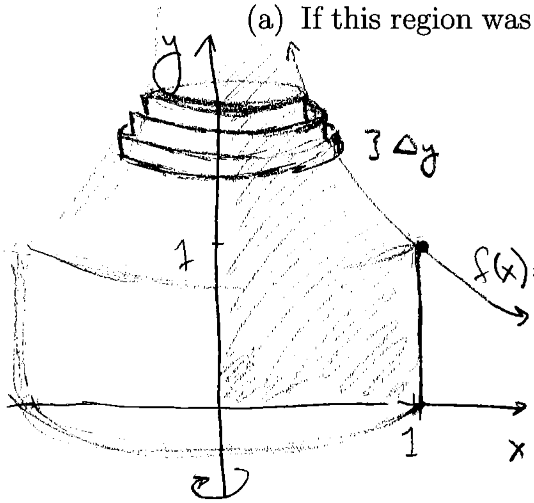
$= \int_0^1 \frac{1}{2}y + \frac{1}{2} - y^{1/2}\, dy$

$= \frac{1}{4}y^2 + \frac{1}{2}y - \frac{2}{3}y^{3/2}\Big]_0^1$

$= \frac{1}{4} + \frac{1}{2} - \frac{2}{3} - 0$

11. Consider the region trapped between $f(x) = \frac{1}{x}$, the x-axis, and from $x = 0$ to $x = 1$.

(a) If this region was revolved about the y-axis, what would the resulting volume be?



horn shape

cylinder

Vol of cylinder + Vol of horn shape

$= \pi 1^2 \cdot 1$ + sum of small cylinders

$= \pi$ + sum of $\pi(\text{radius})^2 \Delta y$

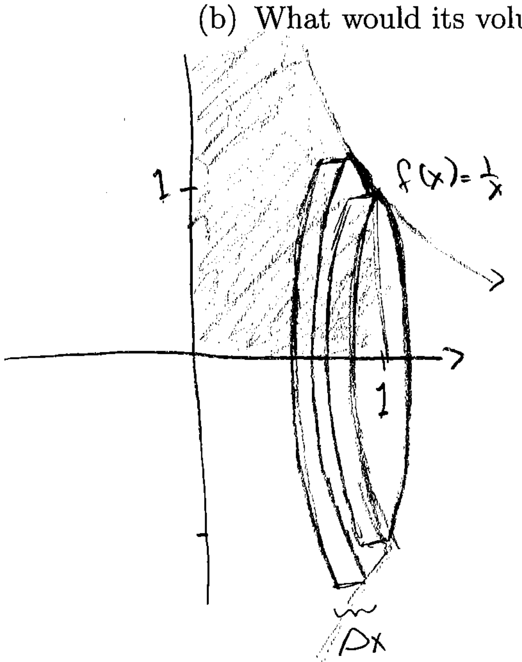
$= \pi$ + sum of $\pi(\text{x-coord})^2 \Delta y$

$= \pi + \int_1^\infty \pi \left(\frac{1}{y}\right)^2 dy$

$= \pi + \pi \lim_{b\to\infty} \int_1^b \frac{1}{y^2}\, dy = \pi + \pi \lim_{b\to\infty} \frac{-1}{y}\Big]_1^b$

$= \pi + \pi\left(0 - \frac{-1}{1}\right) = \pi + \pi = 2\pi$

(b) What would its volume be if it was revolved about the x-axis?



Sum of cylinders

sum of $\pi(\text{radius})^2 \Delta x$

sum of $\pi(\text{y-coord})^2 \Delta x$

$\int_0^1 \pi\left(\frac{1}{x}\right)^2 dx = \lim_{a\to 0^+} \int_a^1 \pi\left(\frac{1}{x}\right)^2 dx$

$= \lim_{a\to 0^+} \pi \frac{-1}{x}\Big]_a^1 = \pi\left[\frac{-1}{1} - \lim_{a\to 0^+} \frac{-1}{a}\right]$ $\to -\infty$

Diverges

12. Spirit accidentally steered off a 7250m cliff on Mars. Thankfully, Spirit was attached to a base station at the top of the cliff by a 1000 meter cable that is .6 kg per meter. How much work does it take to haul Spirit back up to the top of the cliff. Gravity on Mars is $3.69\frac{\text{m}}{\text{s}^2}$ and Spirit has 185kg of mass.

let x be the dist. Spirit has been hauled up, let force function be F

$F(0) = (\text{cable mass} + \text{Spirit mass})\,\text{acceleration}$

$= (1000\text{m} \cdot .6\,\text{kg/m} + 185\text{kg})\, 3.69\,\text{m/s}^2$

$= 2896.65\ \text{N}$

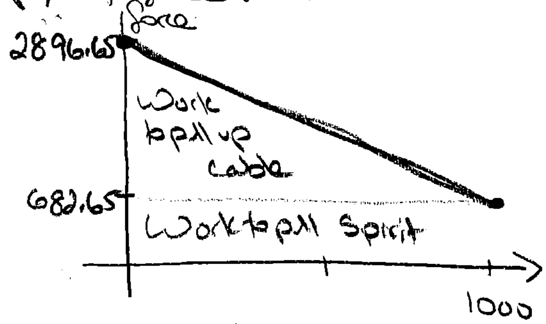
$F(500) = (\tfrac{1}{2}\,\text{cable mass} + \text{Spirit mass})\,\text{acceleration}$

$= (500\text{m} \cdot .6\,\text{kg/m} + 185\,\text{kg}) \cdot 3.69\,\text{m/s}^2$

$= 1789.65\ \text{N}$

$F(999) = (1\text{m of cable} + \text{Spirit mass})\,\text{acceleration}$

$= (.6 + 185) \cdot 3.69\,\text{m/s}^2$



Work is the area trapped below the Force function graphed above

Work for cable + Work for Spirit

$\frac{1}{2}(1000)(2214) + 1000 \cdot 682.65 = 1789650$ Nm

13. The University of Washington, Tacoma has 3000 students. On Monday two students noticed police outside Dr. Card's office and heard Dr. Card was dead. A rumor began to spread and by Tuesday 200 students had heard it. It is reasonable to assume that rate of the spread of the rumor is proportional to the number of possible encounters between students who have heard the rumor and those who have not. Let $y = y(t)$ be the number of students who have heard the rumor after t days. Write a differential equation that describes the above model and solve the differential equation for $P(t)$.

let y be the # of people who heard

$\frac{dy}{dt} = k\, y(3000-y)$

possible encounters between y and 3000 − y is the product

$\Downarrow$

$dt \cdot \frac{\frac{dy}{dt}}{y(3000-y)} = \frac{k\cancel{y(3000-y)}}{\cancel{y(3000-y)}} \cdot dt$

$\frac{1}{y(3000-y)}\,dy = k\,dt$

$\Rightarrow \int \frac{1}{y(3000-y)}\,dy = \int k\,dt$

Partial fractions find A, B so that

$\frac{A}{y} + \frac{B}{3000-y} = \frac{1}{y(3000-y)}$

$A(3000-y) + By = 1$

$3000A + (B-A)y = 1$

$\Rightarrow 3000A = 1 \Rightarrow A = 1/3000$

$B - A = 0 \qquad B = 1/3000$

$\int \frac{1}{3000y}\,dy + \int \frac{1}{3000(3000-y)}\,dy = kt + C \Rightarrow \frac{1}{3000}\ln(y) - \frac{1}{3000}\ln(3000-y) = kt + C$

#6 written up

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

8. [10 each] Evaluate the following if they exist.

(a) $\displaystyle\int_0^{\frac{\pi}{4}} \sec^4 x \tan^4 x \, dx$ $= \displaystyle\int_0^{\pi/4} \tan^4 x (\tan^2 x + 1) \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \displaystyle\int_0^1 u^4(u^2+1)\, du = \int_0^1 u^6 + u^4 \, du$

$= \frac{1}{7}u^7 + \frac{1}{5}u^5 \Big]_0^1$

$= \frac{1}{7} + \frac{1}{5} = \frac{5+7}{35} = \frac{12}{35}$

(b) $\displaystyle\int x \cos^2 x \, dx$ $= \displaystyle\int x \frac{1}{2}[1 + \cos 2x]\, dx = \frac{1}{2}\int x + x\cos 2x \, dx$

$= \frac{1}{2}\left[\int x\, dx + \int x \cos 2x \, dx\right] = \frac{1}{2}\left[\frac{1}{2}x^2 + x\frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x \, dx\right]$

$u = x$ $\quad v = \frac{1}{2}\sin 2x$

$du = dx$ $\quad dv = \cos 2x \, dx$

$= \frac{1}{4}x^2 + x\frac{1}{4}\sin 2x - \frac{1}{4}\int \sin 2x \, dx$

$= \frac{1}{4}x^2 + \frac{1}{4}x\sin 2x + \frac{1}{2}\cdot\frac{1}{2}\cos 2x + c$

(c) $\displaystyle\int_1^\infty \frac{1}{x^2}\, dx$ $= \displaystyle\lim_{t\to\infty}\int_1^t \frac{1}{x^2}\, dx = \lim_{t\to\infty} \frac{-1}{x}\Big]_1^t$

$= \displaystyle\lim_{t\to\infty}\left(\frac{-1}{t} + \frac{+1}{1}\right) = \lim_{t\to\infty}\left[\frac{-1}{t} + 1\right]$

$= \displaystyle\lim_{t\to\infty}\frac{-1}{t} + \lim_{t\to\infty} 1$

$= 0 + 1 = 1$

7. Let $f(x) = \ln x$.

(a) [5] Find the average value of f on the interval $[1, e]$.

$$\frac{1}{e-1}\int^{e} \ln x\, dx = \frac{1}{e-1}\left[(\ln x)\cdot x\Big|_1^e - \int_1^e x\,\frac{1}{x}\,dx\right] = \frac{1}{e-1}\left[x\ln x - x\right]_1^e$$

$$= \frac{1}{e-1}\left[(e\ln e - e) - (1\ln 1 - 1)\right]$$

$u = \ln x$ $\quad v = x$

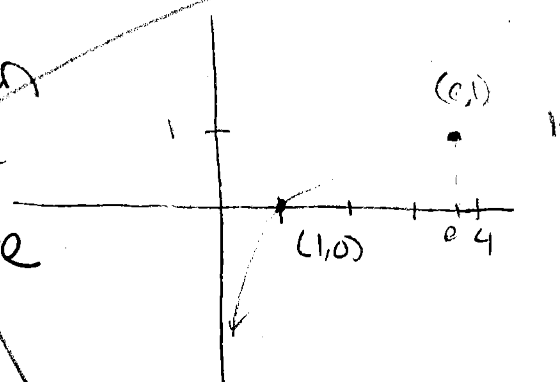
$du = \frac{1}{x}dx$ $\quad dv = dx$

$$= \frac{1}{e-1}\left[(e-e) - (1\cdot 0 - 1)\right]$$

$$= \frac{1}{e-1}\left[1\right] = \frac{1}{e-1}$$

(b) [5] Is there a number c between 1 and e so that $f(c)$ is equal to the value you found in part a? Explain, *briefly* why or why not.

Note $\ln x$ is con't on $[1,e]$. We can thus use the mean value thm which states that there exists a c between 1 & e so that



$$f(c)\cdot[e-1] = \int_1^e \ln x\,dx$$

$$\Rightarrow f(c) = \frac{1}{e-1}\int_1^e \ln x\,dx = \frac{1}{e-1}.$$

Note: we didn't have to find it.

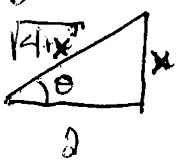
#6 written up cont.

$\sin^2\theta+\cos^2\theta=1$
$\tan^2\theta+1=\sec^2\theta$
$1+\cot^2\theta=\csc^2\theta$

Sohcahtoa

(d) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx = \int \frac{1}{4\tan^2\theta\sqrt{4\tan^2\theta+4}}\cdot 2\sec^2\theta\, d\theta$

$\frac{x}{2}=\tan\theta$ $\qquad x=2\tan\theta$
$\qquad dx=2\sec^2\theta\, d\theta$



$\displaystyle = \int \frac{\cancel{2}\sec^{\cancel{2}}\theta}{4\tan^2\theta\,\cancel{2}\cancel{\sec\theta}}\,d\theta$ for $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$

$\displaystyle = \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{4}\int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\,d\theta$

$\displaystyle = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$, $u=\sin\theta$, $du=\cos\theta\,d\theta$ $\Rightarrow \displaystyle\frac{1}{4}\int\frac{du}{u^2} = \frac{1}{4}\,\frac{-1}{u}+c = \frac{-1}{4\sin\theta}+c$

$\displaystyle = \frac{-1}{4}\csc\theta + c = \frac{-1}{4}\,\frac{\sqrt{4+x^2}}{x}+c$

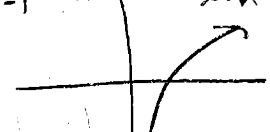
oops

(e) $\displaystyle\int_0^3 \frac{1}{x-1}\,dx$ instead

note $\frac{1}{x-1}$ is not cont at $x=1$

$\displaystyle\int_0^3\frac{1}{x-1}dx = \int_0^1\frac{1}{x-1}dx+\int_1^3\frac{1}{x-1}dx = \lim_{t\to1^-}\int_0^t\frac{1}{x-1}dx+\lim_{s\to1^+}\int_s^3\frac{1}{x-1}dx$

$\ln x$



$\displaystyle = \lim_{t\to1^-}\ln|x-1|\Big]_0^t + \lim_{s\to1^+}\ln|x-1|\Big]_s^3$

$\displaystyle = \underbrace{\lim_{t\to1^-}\left[\ln|t-1|-\cancel{\ln 1}\right]}_{\text{diverges}} + \lim_{s\to1^+}\left[\ln 2-\ln|s-1|\right]$ so $\displaystyle\int_0^3\frac{1}{x-1}dx$ $\boxed{\text{diverges}}$

note $\displaystyle\lim_{t\to1^-}\ln|t-1| = \ln\left(\lim_{t\to1^-}|t-1|\right) \to -\infty$

(f) $\displaystyle\int \frac{17x-1}{2x^2+3x-2}\,dx$

$(2x-1)(x+2)$

$\displaystyle\frac{A}{2x-1}+\frac{B}{x+2}=\frac{17x-1}{(2x-1)(x+2)}$

$u=2x^2+3x-2$
$du=4x+3\,dx$

$\Rightarrow Ax+2A+B2x-B=17x-1$

$\Rightarrow$ $A+2B=17$, $2A-B=-1$ $\Big\}$ $B=2A+1$
$A+2(2A+1)=17 \Rightarrow 5A+2=17$
$\Rightarrow 5A=15$
$A=3$

$\displaystyle\int\frac{17x-1}{(2x-1)(x+2)}dx = \int \frac{3}{2x-1}+\frac{7}{x+2}\,dx = 3\int\frac{1}{2x-1}dx+7\int\frac{1}{x+2}dx$

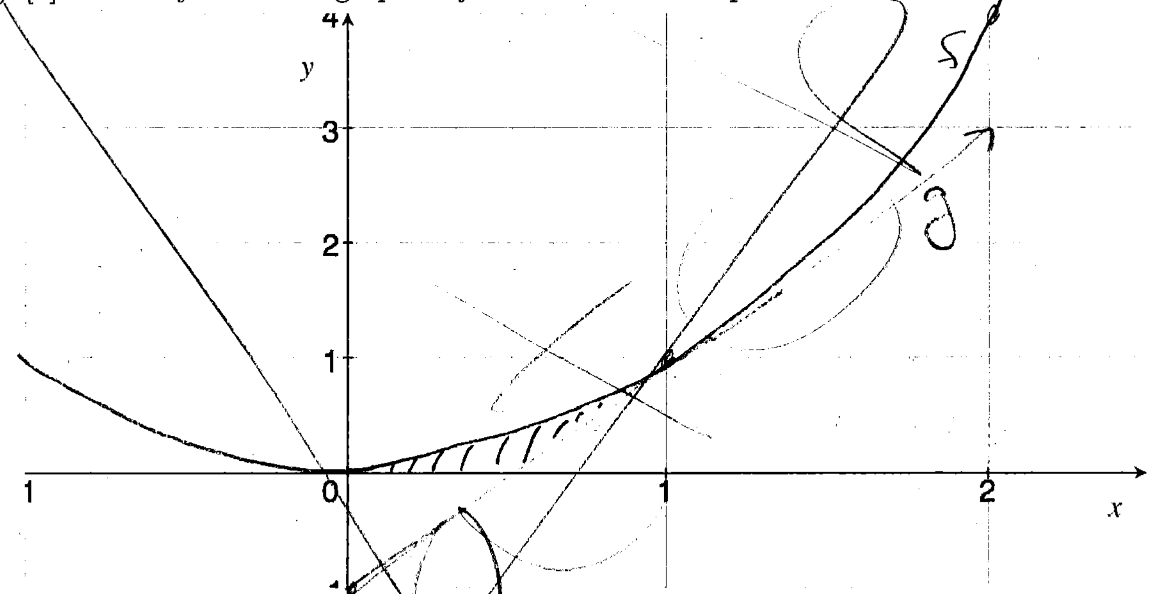
$u=2x-1$
$du=2dx$

$\displaystyle = 3\int\frac{1}{u}\cdot\frac{1}{2}du + 7\ln|x+2|+c$

$\boxed{\displaystyle = \frac{3}{2}\ln|2x-1|+7\ln|x+2|+c}$

9. Let $f(x) = x^2$.

(a) [3] Carefully draw the graph of f on the set of axis provided.



(b) [4] Let g be the function tangent to f at $x = 1$. Find the rule for g and draw the graph of g on the above graph.

looking for m ~~and b~~

$g(x) = mx + b$

$m = f'(1) = 2x\big|_{x=1} = 2\cdot 1 = 2$

$\Rightarrow g(x) = 2x + b$

g passes through the point $(1, f(1)) = (1,1)$

so

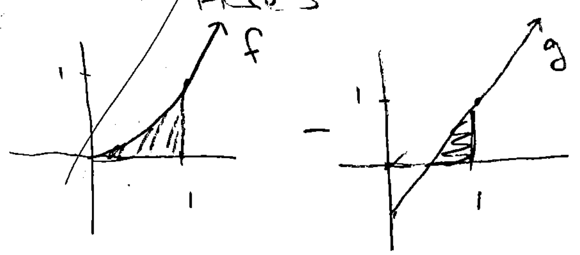
$1 = g(1) = 2(1) + b$

$\Rightarrow 1 = 2 + b \Rightarrow b = -1$

So $\boxed{g(x) = 2x - 1}$

(c) [6] Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x-axis.

I wrote 2 different solutions to this on Midterm 2. Here's another solution one of you came up with:



$$= \int_0^1 x^2\,dx - \tfrac{1}{2}(1)(\tfrac{1}{2})$$

(height)(base)

$$= \tfrac{1}{3}x^3\Big]_0^1 - \tfrac{1}{4} = \tfrac{1}{3} - \tfrac{1}{4} = \tfrac{4-3}{12} = \tfrac{1}{12}$$