Key

EXAM 1 | TMath 125 | PRACTICE

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let a, b, and c be constants. Assume f and g are continuous.

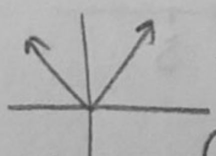
   T F $\int_a^b cf(x)dx = c\int_a^b f(x)\,dx$

   T F $\int f(x)g(x)dx = \int f(x)\,dx \int g(x)dx$

   Consider $\int x \cdot x\,dx$ vs $\int x\,dx \int x\,dx$

   $\frac{1}{3}x^3 + C \neq (\frac{1}{2}x^2)(\frac{1}{2}x^2)+C$

   T F All continuous functions have derivatives.



y=|x| problem @ x=0

   T F All continuous functions have antiderivatives. If you think of antiderivatives as areas this can work!

   T F $\int_{-1}^{1} \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -2$

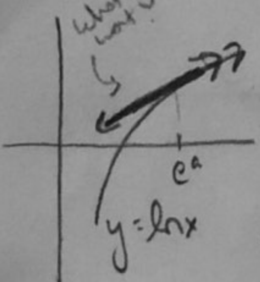
   Area should be positive! Note FTC II can't be used b/c $1/x^2$ is not continuous @ $x=0$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

Favorite Dif. Calc Math 124 Question!

2. Let a be a constant (like 2.5 or something). Find the equation of the line that is tangent to the graph of $y = \ln x$ at $x = e^a$ for some constant a.

looking for $y = mx + b$ or $y - y_1 = m(x - x_1)$



what we want

m = slope of line tangent to y at $x = e^a$

$= y'\big|_{x=e^a}$

$= \frac{1}{x}\Big|_{x=e^a}$

$= \frac{1}{e^a} = e^{-a}$

$\Downarrow$

$m = e^{-a}$

note $y = \ln x \Rightarrow y' = \frac{1}{|x|}$

line passes thru $(e^a, \ln(e^a))$ or (e^a, a)

$\Downarrow$

$y - a = e^{-a}(x - e^a)$

or

$y = e^{-a}x - e^{-a}e^{a} + a$

$= e^{-a}x - 1 + a$

$$f(x) = \begin{cases} 2\sin\left(\frac{\pi}{2}x\right) & \text{if } x < -2 \\ -\sqrt{4-x^2}; & \text{if } -2 \le x \le 2 \\ x-2; & \text{if } 2 < x \end{cases}$$

3. Refer to the above definition of $f(x)$ to answer the following questions.

line w/ slope 1
y intercept @ -2

(a) Carefully graph $f(x)$ from $x = -6$ to $x = 4$.

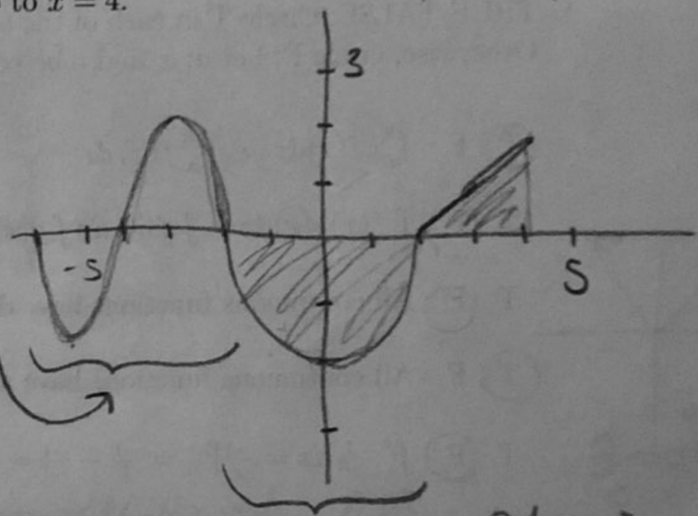
$2\sin\left(\frac{\pi}{2}x\right)$

vert. stretch by 2 ⇒ amplitude of 2

horiz. shrink by $\pi/2$

⇒ period of $\frac{2\pi}{\pi/2} = 2\pi \div \frac{\pi}{2}$

⇒ period of 4



$y = -\sqrt{4-x^2}$ $\quad y \le 0$

$(-y)^2 = 4 - x^2 \Rightarrow x^2 + y^2 = 4$ circle w/ radius 2 centered @ (0,0)

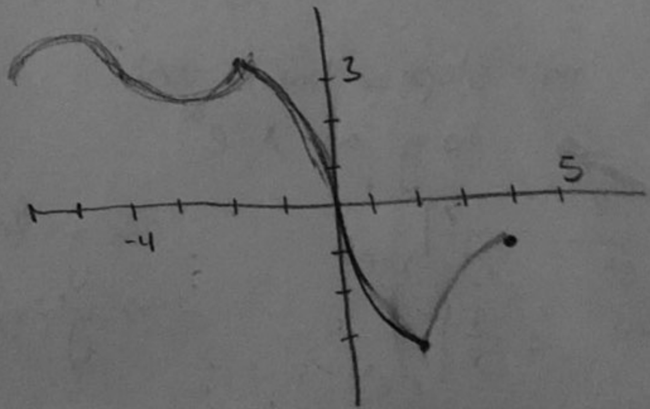
(b) Use your above graph to find $\int_{-2}^{4} f(x)\,dx$.

semicircle area (negative) + triangle area (positive) $= \frac{-1}{2}\pi(2)^2 + \frac{1}{2}\cdot 2\cdot 2$

$= -2\pi + 2$

(c) Give a rough sketch the graph of $\int_0^x f(t)\,dt$ from $x = -6$ to $x = 4$. ? accumulation function?

x	$\int_0^x f(t)dt$
4	$\int_0^4 f(t)dt = -\left(\frac{1}{4}\right)\pi\cdot 2^2 + 2 = 2-\pi$
3	$\int_0^3 f(t)dt = -\left(\frac{1}{4}\right)\pi\cdot 2^2 + \frac{1}{2}$
2	$\int_0^2 f(t)dt = -\left(\frac{1}{4}\right)\pi\cdot 2^2$
0	$\int_0^0 f(t)dt = 0$
-2	$\int_0^{-2} f(t)dt = -\int_{-2}^{0} f(t)dt = -\left(-\frac{1}{4}\pi\cdot 2^2\right)$
-4	$\int_0^{-4} f(t)dt = -\int_{-4}^{0} f(t)dt =$



b/c w/ end points are reversed

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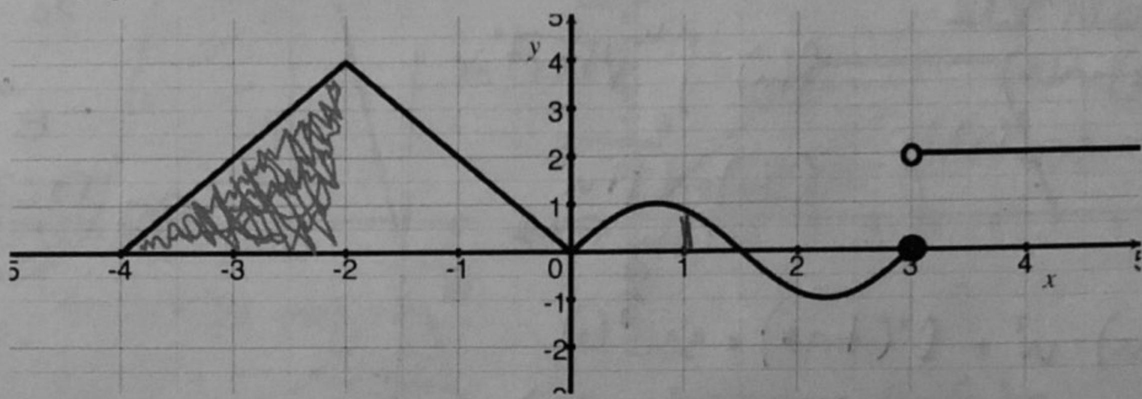
4. For this page you will use the function f graphed below and the function g. It is given that $\int_{-4}^{-2} g(x)\,dx = 3$ and $\int_{-2}^{0} g(x)\,dx = 2$



(a) Find $\int_{1}^{1} f(x)\,dx$

0

(b) Describe the shaded area as a definite integral.

$\int_{-4}^{-2} f(t)\,dt$

(c) Find $\int_{-4}^{0} f(x)\,dx$

a triangle w/ height 4, base 4

$\Rightarrow \frac{1}{2} \cdot 4 \cdot 4 = 8$

(d) Find $\int_{-4}^{0} f(x) + g(x)\,dx$

$$= \underbrace{\int_{-4}^{0} f(x)\,dx}_{\substack{8 \\ \text{from (c)}}} + \underbrace{\int_{-4}^{0} g(x)\,dx}$$

$$+ \underbrace{\int_{-4}^{-2} g(x)\,dx}_{3} + \underbrace{\int_{-2}^{0} g(x)\,dx}_{2} = 13$$

3   3 + 2

from top of page

5. Find

$\frac{d}{dx}\int_0^{\tan x}\sqrt{1+r^3}\,dr$

Chain Rule

$u = \tan(x)$ $\quad f(u) = \int_0^u \sqrt{1+r^3}\,dr$

$u' = \sec^2(x)$

$f'(u) = \sqrt{1+u^3}$

↳ by FTC I

$f'(u)\cdot u' = f'(\tan x)\cdot \sec^2(x)$

$= \sqrt{1+(\tan x)^3}\cdot \sec^2(x)$

$= \sqrt{1+\tan^3(x)}\cdot \sec^2(x)$

$\left(\int_x^0 e^{\arctan(t)}\,dt\right)' = -\int_0^x e^{\arctan(t)}\,dt$

$= -e^{\arctan(x)}$

by FTC I

6. Evaluate the following.

$\int \frac{t^3-3t^2}{2t}\,dt$ (family of functions)

$= \int \frac{t^3}{2t} - \frac{3t^2}{2t}\,dt = \int \frac{t^2}{2} - \frac{3}{2}t\,dt$

$= \int \frac{t^2}{2}\,dt - \int \frac{3}{2}t\,dt$

$= \frac{1}{2}\frac{1}{3}\cdot t^3 - \frac{3}{2}\cdot\frac{1}{2}t^2 + C$

$= \frac{1}{6}t^3 - \frac{3}{4}t^2 + C$

Ck: $\frac{d}{dt}\left(\frac{1}{6}t^3 - \frac{3}{4}t^2 + C\right)$

$= \frac{1}{6}\cdot 3t^2 - \frac{3}{4}\cdot 2t + 0$

$= \frac{1}{2}t^2 - \frac{3}{2}t$ ✓

$\int_4^7 x^3\sqrt{x^2+1}\,dx$ (a number)

$u = x^2+1 \Rightarrow u-1 = x^2$

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$\int_4^7 x^2\sqrt{x^2+1}\,x\,dx = \int_{4^2+1}^{7^2+1} (u-1)\sqrt{u}\,\frac{1}{2}\,du$

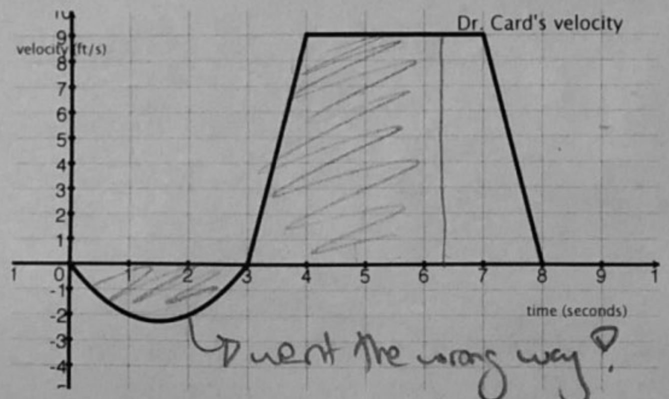
$= \int_{17}^{50} \frac{1}{2}(u-1)u^{1/2}\,du = \frac{1}{2}\int_{17}^{50}(u-1)u^{1/2}\,du$

$= \frac{1}{2}\int_{17}^{50} u^{3/2} - u^{1/2}\,du$

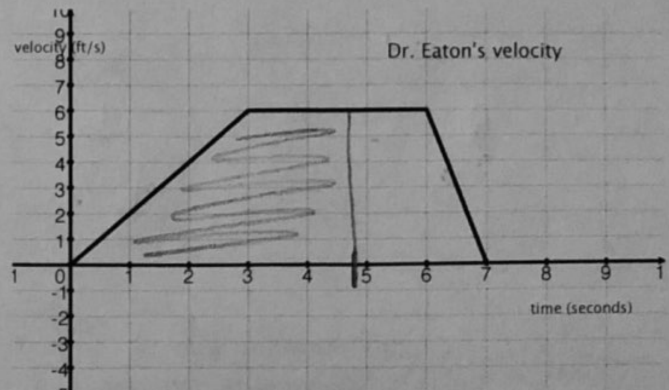
$= \frac{1}{2}\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big]_{17}^{50}$

$= \left(\frac{1}{5}50^{5/2} - \frac{2}{3}50^{3/2}\right) - \left(\frac{1}{5}\cdot 17^{5/2} - \frac{2}{3}\cdot 17^{3/2}\right)$

7. Dr. Card and Dr. Eaton decide to have a short race. The following is a graph of their respective *velocities* at time t measured in seconds.



went the wrong way?



(a) Estimate the total distance each one runs during the race.

Dr Card

$|5| + \frac{1}{2}(1)(9) + 3\cdot 9 + \frac{1}{2}(1)\cdot 9 = 41\text{ ft}$

Dr Eaton:

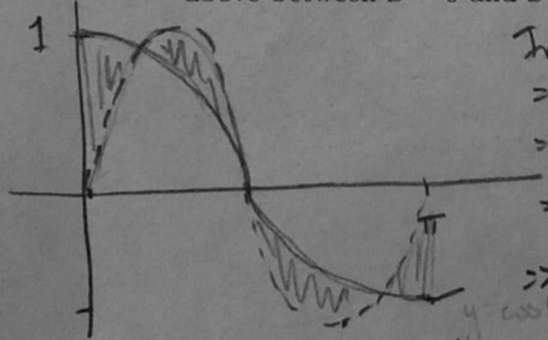
$\frac{1}{2}\cdot 3\cdot 6 + 3\cdot 6 + \frac{1}{2}\cdot 1\cdot 6$

$9 + 18 + 3 = 30\text{ ft}$

(b) If the race is 20 ft, who wins the race? Explain how you know.

Dr Eaton. Dr Eaton accumulated about 20 ft right befor 5 sec where as Dr Card had to make up for his wrong direction & then get another 20 ft about 6 sec => Dr. Eaton won.

8. Consider $y = \sin(2x)$ and $y = \cos(x)$. Find the area of the region bounded by the above between $x = 0$ and $x = \pi$.



Intersections when $\sin(2x) = \cos(x)$

$\Rightarrow \sin(2x) - \cos(x) = 0$

$\Rightarrow 2\cos x \sin(x) - \cos(x) = 0$ double angle id

$\Rightarrow \cos(x)(2\sin(x) - 1) = 0$

$\Rightarrow \cos(x) = 0$ or $2\sin(x) - 1 = 0$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\frac{\pi}{6}, \frac{5\pi}{6}$

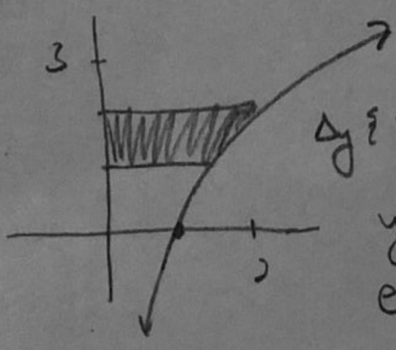
and coterminal angles

$$\int_0^{\pi/6} \cos x - \sin(2x)\,dx + \int_{\pi/6}^{\pi/2} \sin 2x - \cos(x)\,dx + \int_{\pi/2}^{5\pi/6} \cos(x) - \sin 2x\,dx + \int_{5\pi/6}^{\pi} \sin 2x - \cos(x)\,dx$$

$$= \left[\sin x + \tfrac{1}{2}\cos(2x)\right]_0^{\pi/6} + \left[-\tfrac{1}{2}\cos(2x) - \sin x\right]_{\pi/6}^{\pi/2} + \left[\sin x + \tfrac{1}{2}\cos(2x)\right]_{\pi/2}^{5\pi/6} + \left[-\tfrac{1}{2}\cos(2x) - \sin x\right]_{5\pi/6}^{\pi}$$

$$= \left(\tfrac{1}{2} + \tfrac{1}{2}\cdot\tfrac{1}{2}\right) - \left(0 + \tfrac{1}{2}\right) + \left(-\tfrac{1}{2}(-1) - 1\right) - \left(-\tfrac{1}{2}\cdot\tfrac{1}{2} - \tfrac{1}{2}\right) + \left(\tfrac{1}{2} + \tfrac{1}{2}\cdot\tfrac{1}{2}\right) - \left(1 + \tfrac{1}{2}(-1)\right) + \left(-\tfrac{1}{2}\cdot 1 - 0\right) - \left(-\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\right)$$

9. Find the area bounded by $y = \ln x$, $y = 1$, $y = 2$, $x = 0$.



$\Delta y \approx$

$y = \ln x$

$e^y = x$

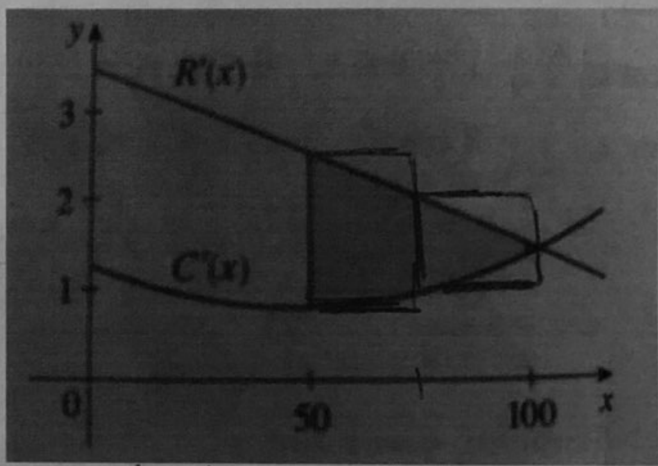
$$\int_1^2 (\text{x-value})\,dy = \int_1^2 e^y\,dy = e^y\Big]_1^2 = e^2 - e^1$$

10. Expand $\sum_{j=2}^{6} \left(\frac{j}{j^2 - 3} \right)$. (You do *not* need to compute this!)

$j=2 \quad j=3 \quad j=4 \quad j=5 \quad j=6$

$$\frac{2}{2^2-3} + \frac{3}{3^2-3} + \frac{4}{4^2-3} + \frac{5}{5^2-3} + \frac{6}{6^2-3}$$

11. The figure shows graphs of the marginal revenue function R' and the marginal cost function C'. for a manufacturer. Let $R(x)$ and $C(x)$ represent the revenue and cost when x units are manufactured respectively. Assume that R and C are measured in thousands of dollars.



(a) What is the meaning of the area of the shaded region?

Profit made by producing 50 more units after an initial 50 units

(b) Use two left-hand approximating rectangles to estimate the shaded region.

$25(2.5-.8)+25(2.1-1)$

(c) Classify if your approximation is an over or underestimate.

overestimate