EXAM 1 # TMath 125 PRACTICE

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let a, b, and c be constants. Assume f and g are continuous.

   T F $\int_a^b cf(x)dx = c\int_a^b f(x)\,dx$

   T F $\int f(x)g(x)dx = \int f(x)\,dx \int g(x)dx$

   T F All continuous functions have derivatives.

   T F All continuous functions have antiderivatives.

   T F $\int_{-1}^{1} \frac{1}{x^2}\,dx = \frac{-1}{x}|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -2$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. Let a be a constant (like 2.5 or something). Find the equation of the line that is tangent to the graph of $y = \ln x$ at $x = e^a$ for some constant a.

$$f(x) = \begin{cases} 2\sin\left(\frac{\pi}{2}x\right) & \text{if } x < -2 \\ -\sqrt{4-x^2}; & \text{if } -2 \le x \le 2 \\ x - 2; & \text{if } 2 < x \end{cases}$$

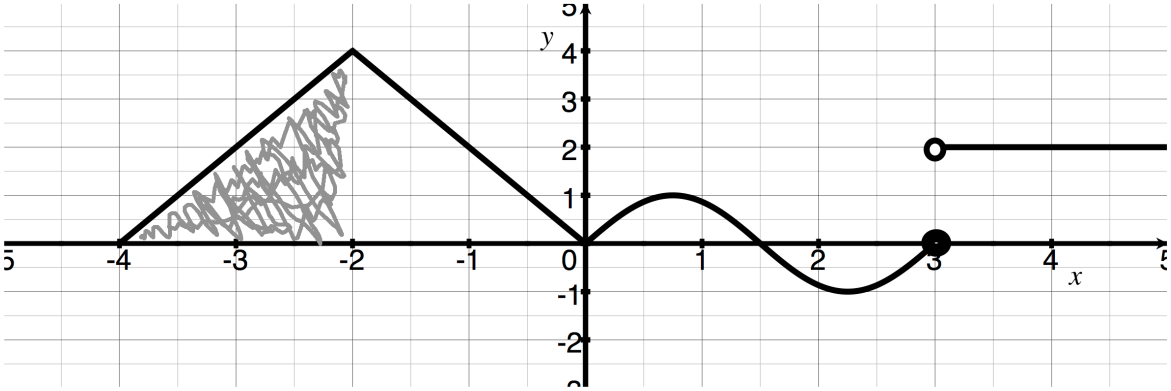
3. Refer to the above definition of $f(x)$ to answer the following questions.

   (a) Carefully graph $f(x)$ from $x = -6$ to $x = 4$.

   (b) Use your above graph to find $\int_{-2}^{4} f(x)\,dx$.

   (c) Give a rough sketch the graph of $\int_{0}^{x} f(t)\,dt$ from $x = -6$ to $x = 4$.

4. For this page you will use the function f graphed below and the function g. It is given that $\int_{-4}^{-2} g(x)\,dx = 3$ and $\int_{-2}^{0} g(x)\,dx = 2$



(a) Find $\int_{1}^{1} f(x)\,dx$

(b) Describe the shaded area as a definite integral.

(c) Find $\int_{-4}^{0} f(x)\,dx$

(d) Find $\int_{-4}^{0} f(x) + g(x)\,dx$

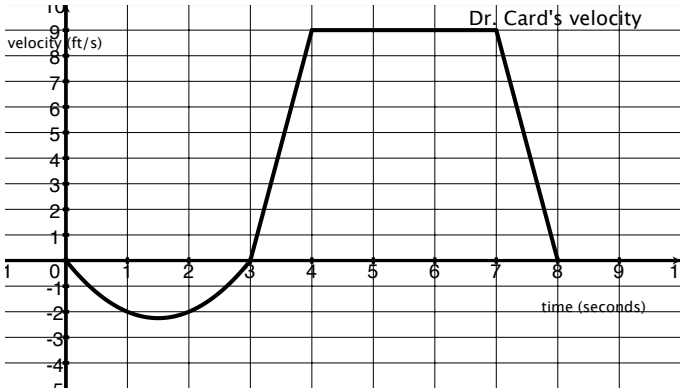
5. Find

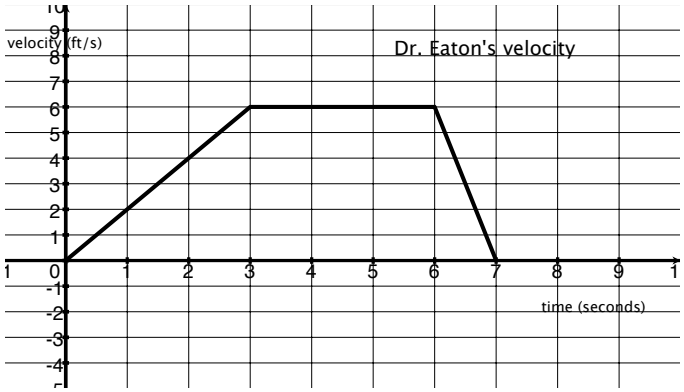
$\frac{d}{dx}\int_0^{\tan x}\sqrt{1+r^3}\,dr$ $\qquad\qquad$ $\left(\int_x^0 e^{\arctan(t)}\,dt\right)'$

6. Evaluate the following.

$\int \frac{t^3-3t^2}{2t}\,dt$ $\qquad\qquad$ $\int_4^7 x^3\sqrt{x^2+1}\,dx$

7. Dr. Card and Dr. Eaton decide to have a short race. The following is a graph of their respective *velocities* at time t measured in seconds.





(a) Estimate the total distance each one runs during the race.

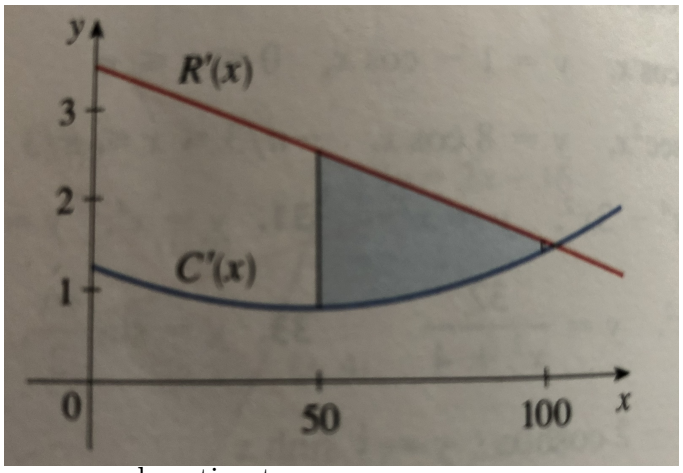
(b) If the race is 20 ft, who wins the race? Explain how you know.

8. Consider $y = \sin(2x)$ and $y = \cos(x)$. Find the area of the region bounded by the above between $x = 0$ and $x = \pi$.

9. Find the area bounded by $y = \ln x$, $y = 1$, $y = 2$, $x = 0$.

10. Expand $\sum_{j=2}^{6} \left(\frac{j}{j^2 - 3} \right)$. (You do *not* need to compute this!)

11. The figure shows graphs of the marginal revenue function R' and the marginal cost function C'. for a manufacturer. Let $R(x)$ and $C(x)$ represent the revenue and cost when x units are manufactured respectively. Assume that R and C are measured in thousands of dollars.



(a) What is the meaning of the area of the shaded region?

(b) Use two left-hand approximating rectangles to estimate the shaded region.

(c) Classify if your approximation is an over or underestimate.