Exam 2

TMath 125

Autumn 2023

Expression	Substitution	Restrictions	Reason
$\sqrt{a^2 - x^2}$	$x = a\sin(\theta)$	$\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$	$1 - \sin^2(\theta) = \cos^2(\theta)$
	OR $x = a\cos(\theta)$	$0 \le \theta \le \pi$	$1 - \cos^2(\theta) = \sin^2(\theta)$
$\sqrt{a^2 + x^2}$	$x = a\tan(\theta)$	$\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$	$1 + \tan^2(\theta) = \sec^2(\theta)$
$\sqrt{x^2 - a^2}$	$x = a\sec(\theta)$	$0 \le \theta \le \pi$	$\sec^2(\theta) - 1 = \tan^2(\theta)$

Reasonable supporting work must be shown to earn credit.

1. Each of the following is wrong. Explain why.

   (a) [2] (10/23 Activity #1) $\int 2x\sin(x)\,dx = -x^2\cos(x) + c$

   (b) [2] (WebHW7-3 #2) We let $x = \sqrt{7}\sec(\theta)$, then $\int \frac{x^2}{\sqrt{x^2-7}}\,dx = \frac{7\sec^2(\theta)}{\sqrt{7\sec^2(\theta) - 7}} \cdot \tan(\theta)\sec(\theta)$

   (c) [2] (PracticeExam#1) Substitution $\int_0^1 y(y^2+1)^5\,dy = \int_0^1 \frac{1}{2}u^5\,du$

   (d) [2] The region bounded by $y = \frac{1}{2}x + 1$, $y = 0$, $x = 0$, and $x = 2$, revolved around the line $y = -1$ has volume $\int_0^2 \pi\left(\frac{1}{2}x + 1\right)^2 dx$

2. [4] (Quiz4 #1) Describe the strategy for evaluating $\int \cot^m(x)\csc^n(x)\,dx$ when m is odd and both $m, n > 0$. Consider the two worked out examples on the right.

$$\int \cot^3(x)\csc(x)\,dx$$
$$= \int \cot^2(x)\,\cot(x)\csc(x)\,dx$$
$u = \csc(x)$

$du = (\csc(x))' = \left(\frac{1}{\sin x}\right)' dx$

$= \frac{\sin(x)\cdot 0 - 1\cos x}{\sin^2 x}\,dx$

$= -\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}\,dx$

$= -\cot(x)\csc(x)\,dx$

$\Rightarrow -du = \cot(x)\csc(x)\,dx$

Recall $\frac{\sin^2 x + \cos^2 x = 1}{\sin^2 x \qquad \sin^2 x}$

$\Rightarrow 1 + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\Rightarrow 1 + \cot^2 x = \csc^2 x$

$\Rightarrow \cot^2(x) = \csc^2 x - 1$

$$\int \cot^2(x)(-du) = \int (\csc^2 x - 1)(-du)$$
$$= -\int u^2 - 1\,du = -\left(\frac{1}{3}u^3 - u\right) + C$$
$$= -\frac{1}{3}\csc^3(x) + \csc(x) + C$$
CK: $\left[-\frac{1}{3}\csc^3(x) + \csc(x) + C\right]' = \frac{+3}{3}\csc^2(x)(+\cot(x)\csc(x)) - \csc(x)\cot(x)$ ✓

$$\int \cot(x)\csc^3(x)\,dx$$
$$= \int \csc^2(x)\,\cot(x)\csc(x)\,dx$$
$u = \csc(x)$

$du = -\cot(x)\csc(x)\,dx$

$\Rightarrow -du = \cot(x)\csc(x)\,dx$

$$\int u^2(-du) = -\int u^2\,du$$
$$= -\frac{1}{3}u^3 + C$$
$$= -\frac{1}{3}\csc^3(x) + C$$
CK: $\left[-\frac{1}{3}\csc^3(x) + C\right]' = \frac{+1}{3}\cdot 3\csc^2(x)(\cot(x)\csc(x))$ ✓

3. For each of the following, identify the technique you would use to find the indefinite integral. For example, if you think substitution would work, write “substitution” and identify what u would be. If you think integration by parts, write “integration by parts” and identify what u and dv would be.

   (a) [2]

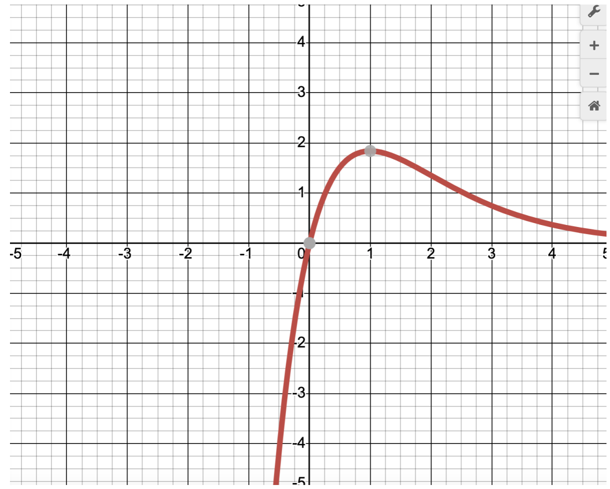
$$\int 4\sin^2(t)\cos^3(t)\,dt$$

   (b) [2]

$$\int \frac{y}{\sqrt{4y^2+9}}\,dy$$

4. [4] Evaluate *one* of the indefinite integrals above.

5. A particle is moving along a straight line and has a velocity $v(t) = 5te^{-t}$ meters per second after t seconds.



(a) [1] (Quiz4#3) Find the velocity of the particle when $t = 2$.

(b) [3] (WordProblem2#1) Find the acceleration of the particle when $t = 2$.

(c) [3] (WrittenHW7.1#75) Find the expression that you could give to technology that would return the change in distance in the first 2 seconds.

6. Let f be a twice differentiable function with the values f and f' given below.

x	$f(x)$	$f'(x)$
1	-2	4
5	3	5

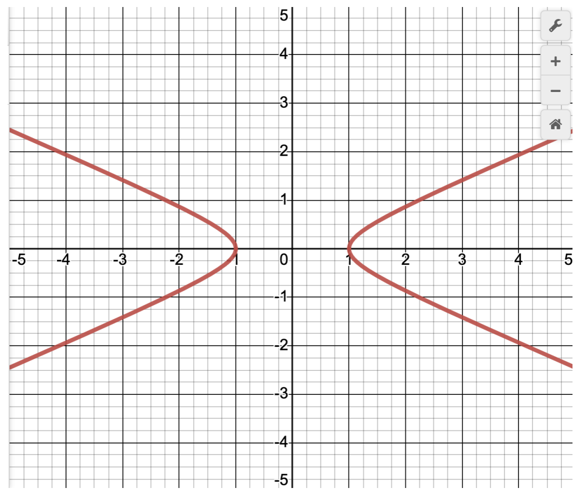
(a) [1] Find $f(5)$, if possible.

(b) [3] (PracticeExam2#4) Evaluate $\int_1^5 f''(x)\,dx$, if possible.

(c) [3] (Quiz4#2) Evaluate $\int_1^5 6xf''(x)\,dx$, if possible.

7. [3] One problem substituted $x = 6\tan(\theta)$ and then integrated to get $\frac{1}{36}\sin(\theta) + C$. Convert this answer (that is currently in terms of θ) back into terms of x.

8. The graph of $x^2 - 4y^2 = 1$ is given below.



(a) [2] Shade the region trapped by $x^2 - 4y^2 = 1$, $y = 2$, and the x-axis.

(b) [4] (WrittenHW§7.3#40) Set up the definite integral (but do *not* compute!) that will find the area of the shaded region.

(c) [2] (§6.2#2) Consider the volume whose base was shaded in (a). The cross sections (perpendicular to the y axis) of the object are squares. Sketch one of the approximating cylinders of this volume.

(d) [4] (WordProblem2#7) Set up the definite integral that would find the volume described above.

9. [1] Provide a topic or problem that you studied for but did not see on the exam.