Ave 76%

Typos #1 horiz axis is in seconds
6a) $f(x)\,dx$
3b) dt not dx

Key

Exam 1

TMath 125

Autumn 2023

4 sale: equation of circle (.5pt)

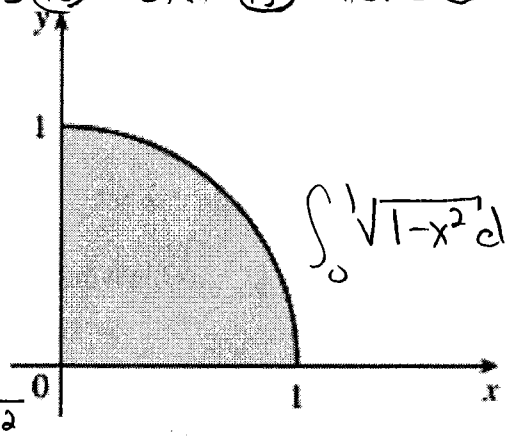
Show all your work.
Reasonable supporting work must be shown to earn credit.

1. [3] (SummationActivity #1) Expand $\sum_{i=1}^{4}\left(\frac{2i}{i+1}\right)$.
(You do *not* need to compute or simplify this!)

$$\frac{2\cdot1}{1+1}+\frac{2(2)}{2+1}+\frac{2(3)}{3+1}+\frac{2(4)}{4+1}$$

start @ i=1 (+.5) end at i=4 (+.5) sums (+.5) start (+.5) terms (+1)

2. [4] (Quiz1#1) Consider the shaded area trapped between the circle centered at $(0,0)$ with radius one and the x and y axis shown below. Display the shaded area as a definite integral. Make sure you write it in such a way that technology could compute your answer.



$$\int_0^1 \sqrt{1-x^2}\,dx$$

start (+.5) bounds (+1) notation/dx (+1) circle formula (+1) integral (+.5)

Circle centered at (h,k) w/ radius r
$\Rightarrow (x-h)^2+(y-k)^2=r^2$
here $(h,k)=(0,0)$ and $r=1$ so
$x^2+y^2=1 \Rightarrow y^2=1-x^2 \Rightarrow y=\sqrt{1-x^2}$

3. [6] (WebHW5-5#8 & WebHW5-3#16) Outline how you would find the following. For example, if you use substitution, identify the u and du that an be used to find an antiderivative. If you use a theorem or rule, identify which one. You do *not* need to integrate!!

(a) $\int \frac{(\arctan(x))^4}{x^2+1}\,dx$

substitution?

$u=\arctan(x)$

$du=\frac{1}{1+x^2}\,dx$

$(1+x^2)\,du=dx$

(+.5 try some method) (+1 tried correctly) got it (+1.5)

$$\int \frac{u^4}{x^2+1}(1+x^2)\,du=\int u^4\,du \Rightarrow \frac{1}{5}u^5+C \text{ or } \frac{1}{5}(\arctan(x))^5+C$$

(b) $\frac{d}{dx}\left(\int_2^{3x+5}\frac{t}{1+t^2}\,dt\right)=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{d}{du}\left(\int_2^{u}\frac{t}{1+t^2}\,dt\right)\cdot\frac{du}{dx}$

FTCI and chain rule?

try something (+.5) tried correctly (+1)

$u=3x+5$

$du/dx=3$

got it (+1.5)

FTCI

$$\frac{u}{1+u^2}\cdot 3=\frac{3x+5}{1+(3x+5)^2}\cdot 3$$

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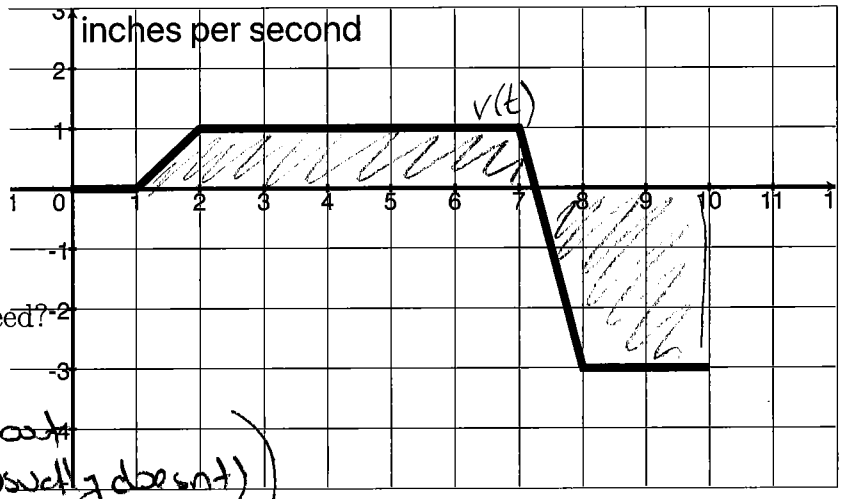
4. Let $v(t)$ be the piece-wise defined function graphed below that is comprised of straight lines. The graph of v reports the velocity (inches per second) of a bug walking along the top of wooden gate at time t. At $t = 0$, the bug is in the middle of the gate and the positive direction be movement to the right.

Let $p(x) = \int_0^x v(t)\,dt$



(a) [1] (Quiz2#1)
Estimate $v(4)$.

1 inch/sec

(b) [2] (WordProblem#12)
What is the bug's top speed?

(+1) 3 inches/sec

start (+.5)
units/sense (+.5)

(unless you care about direction (speed usually doesn't) then 1 inch/sec)

(c) [2] (WebHW5-3#1) Find $v'(6)$.

derivative of $v(t)$ (+.5) at $t=6$ (+.5) ... slope of tang. line to v@ 6 (+.5)

$= 0$ inch/sec^2 (+.5)

(d) [2] (WordProblems#1) Interpret $v'(6)$ in terms of distance, velocity, or acceleration of the bug.

start (+.5)
acc is zero (+1)
vel is const

The acceleration of the bug at 6 seconds is zero (+.5)
or
The velocity of the bug at 6 seconds is constant

(e) [2] (WrittenHW5-3#4) Estimate $p(10)$.

area (+.5)
negative area (+.5)
split areas (+.5)
get it (+.5)

$p(10) = \int_0^{10} v(t)\,dt = \frac{1}{2}\cdot 1\cdot 1 + 5\cdot 1 + \frac{1}{2}\cdot\frac{1}{4}\cdot 1 - \frac{1}{2}\cdot\frac{3}{4}\cdot 3 - 2\cdot 3$

$= \frac{1}{2} + 5 + \frac{1}{8} - \frac{9}{8} - 6 = -1 + \frac{4+1-9}{8} = -1 + \frac{-4}{8}$

$= -1 - \frac{1}{2}$ or $-\frac{3}{2}$ inches

(f) [2] (IndefinitieActivity #3) How far from the middle is the bug after 10 seconds?

start (+.5)
same as (e) (+.5)
sense (+.5)

The net distance traveled was $-3/2$ inches
or $3/2$ inches to the left of the center

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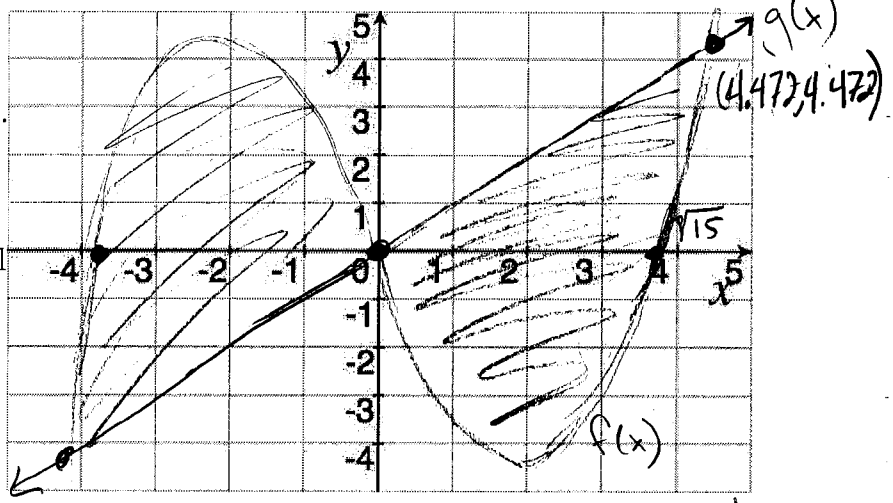
$0 = \frac{1}{5}x^2 - 3$

$3 = \frac{1}{5}x^2$

$\pm\sqrt{15} = x$

$= x\left(\frac{1}{5}x^2 - 3\right)$

5. (WebHW6-1#3) Consider the area trapped by $f(x) = \frac{1}{5}x^3 - 3x$, and $g(x) = x$.



(a) [2] Sketch and shade the region bounded by the curves.

f(x) (+1) region (+.5)

g(x) (+.5)

(b) [4] Set up the definite integral (but do *not* compute!)that will find the area of the shaded region.

(+.5) limits of integrals

found bounds (+1)

broke into 2 integrals (+1)

parenthesis (+.5)

integrands (+1)

$$\int_{-4.472}^{0} \left(\frac{1}{5}x^3 - 3x\right) - x\,dx + \int_{0}^{4.472} x - \left(\frac{1}{5}x^3 - 3x\right) dy$$

f(x) − g(x) g(x) − f(x)

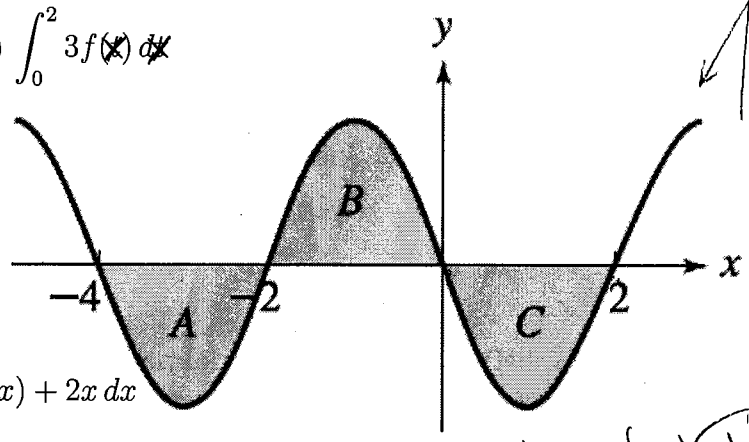
$x = \frac{1}{5}x^3 - 3x$

$0 = \frac{1}{5}x^3 - 4x$

$0 = x\left(\frac{1}{5}x^2 - 4\right)$

$x = 0, \quad \frac{\pm\sqrt{+(\frac{1}{5})4 \cdot 4}}{2\cdot\frac{1}{5}} = \frac{10}{\sqrt{5}} = 2\sqrt{5}$

6. Each of the regions, A, B, & C bounded by the graph of f and the x axis has area 5. Find the following:



(a) [2] (DefiniteIntegralActivity#3) $\int_0^2 3f(x)\,dx$

$3\int_0^2 f(x)\,dx$ (+.5)

sketch (+.5)

$3(-5) = -15$

neg (+.5) value (+.5)

(b) [4] (WebHW5-2&5-3#7) $\int_{-4}^{2} f(x) + 2x\,dx$

properties of int (+1.5)

$$\int_{-4}^{2} f(x) + 2x\,dx = \int_{-4}^{2} f(x)\,dx + \int_{-4}^{2} 2x\,dy$$

$$= \int_{-4}^{-2} f(x)\,dx + \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + x^2\Big]_{-4}^{2}$$

(+.5)

$$= \underbrace{-5 + 5 - 5}_{(+1)} + [4 - 16]$$

FTC (+1)

$$= -5 - 12 = -17$$

12

24
26
=> 50 points

7. [4] Each of the following is wrong. Explain why.

$u = \cos(x)$
$du = -\sin(x)\,dx$ | also sign error(?)

(a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin(x)}{\cos^2(x)}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{u^2}\,du = -u^{-1}\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \left(-\frac{\pi}{3}\right)^{-1} - \left(-\frac{\pi^{-1}}{6}\right) = \frac{-3}{\pi} + \frac{6}{\pi} = \frac{3}{\pi}$

true/did not misid (+.5)
sense (+.5)
bounds (+1)

↳ need to change the limits to be with respect to u

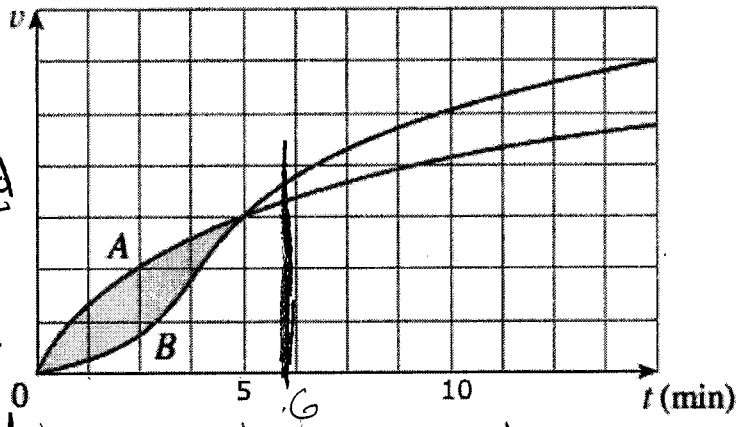
for example, the lower bound $x = \pi/6$ would become $u = \cos(\pi/6)$ or $\sqrt{3}/2$

(b) $\int_1^2 \frac{4}{x^3}\,dx = \int_1^2 4x^{-3}\,dx = 4(-3)x^{-4}\Big|_1^2 = -12 \cdot 2^{-4} - (-12 \cdot (1)^{-4}) = -192 + 12 = 180$

↳ took derivative instead of antiderivative

true/did not misID (.5)
sense (+.5)
bounds (+1)

8. (WebHW6-1#9) Two cars A and B start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.



(a) [2] Explain the meaning of the intersection.

start (+.5)
sense (+.5)

The cars have the same velocity/speed (+1)

(b) [3] Which car is leading after 6 minutes? Justify your answer.

(+1) Car A, not car B
(+1) is going faster but car A has covered more ground as seen by the shaded region
(+.5) start reason
(+.5) sense

9. (WebHW5.5 #13) A bacterial population starts with 300 bacteria and grows at a rate of $r(t) = (450.268)e^{1.12567t}$ bacteria per hour. How many bacteria will there be after three hours?

(a) [2] How many bacteria will there be after three hours?

total population = initial population (+.5) + net change

$= 300 + \int_0^3 (450.268)e^{1.12567t}\,dt$ (+1.5)

$\approx 11{,}613$ (+.5)

Notation (.5)

(b) [3] Write down a function that returns how many bacteria there are at time t.

bacteria at time t $= \underbrace{300}_{+.5} + \int_0^t 450.268e^{1.12567x}\,dx$ (+1.5) works

function (+.5)

start (+.5)

$= 300 + \frac{450.268}{1.12567}e^{1.12567x}\Big]_0^t$

$u = 1.12567x$
$du = 1.12567\,dx$

$= 300 + 400e^{1.12567x} - 400$

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