Eric Guo

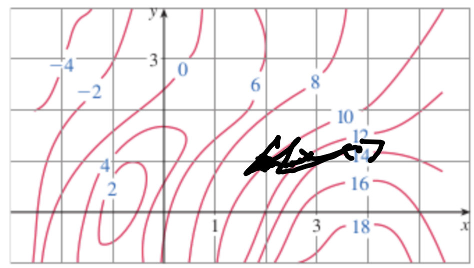
Exam 1

TMath 126

Autumn 2023

1. [12] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

(a) (Suggested §14.3#6) A contour map is given for a function f below. This map implies $f_x(2,1) \approx -2$.



False, then $f_x(2,1)$ would be $\sim +2$ not -2

(b) (dotActivity#1) If $\overrightarrow{w}$ and $\overrightarrow{v}$ are vectors in 3D, then $(\overrightarrow{w} \cdot \overrightarrow{j}) + \overrightarrow{v}$ returns a vector.

False, because $\vec{w} \cdot \vec{j}$ results in a scalar.

(c) (WebHW14.2#2) The limit $\lim_{(x,y)\to(\frac{3\pi}{2},\pi)} y\sin(x-y) = \pi$

True, double check [illegible]

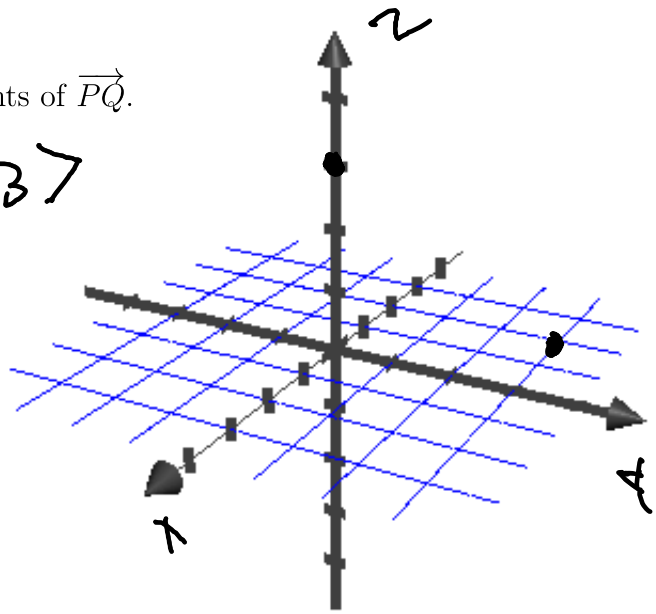
(d) (§13.2#26) If $\overrightarrow{r}(t) = \langle 2^t, \ln(t+1), t\rangle$, then the line tangent to $\overrightarrow{r}(0)$ is:

$$\langle 1,0,0\rangle + \langle 2^t \ln(2), \frac{1}{1+t}, 1\rangle$$

True, the deriv of $2^t = 2^t \ln(2)$, $\ln(t+1) = 1/1+t$ and $t = 1$, and we $r=0$, we get $\langle 1,0,0\rangle$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider the points $P(0, 0, 3)$ and $Q(-2, 3, 0)$



(a) [1] (PracticeExam1#2) Find the components of $\overrightarrow{PQ}$.

$\langle -2-0, 3-0, 0-3 \rangle$

$\boxed{\langle -2, 3, -3 \rangle}$

(b) [2] (DotActivity#2)
Find a vector parallel to $\overrightarrow{PQ}$.

$\boxed{\langle -6, 9, -9 \rangle}$

(c) [3] (Quiz2#1) Find the angle $\overrightarrow{PQ}$ makes with $\langle 0, 1, 3 \rangle$.

$A \times B = \|A\| \times |B| \times \cos(\theta)$

$\langle -2, 3, -3 \rangle \cdot (0, 1, 3) = \langle -2, 3, -3 \rangle \times (0, 1, 3) \times \cos(\theta)$

$\sqrt{-2^2 + 3^2 + -3^2} = \sqrt{22}$

$-2 \times 0 \times 3 \times 1 + 3 \times -3 \Rightarrow -6 \quad \sqrt{0^2 + 1^2 + 3^2} = \sqrt{10}$

$-6 = \sqrt{22} \times \sqrt{10} \times \cos(\theta)$ $\quad \theta = 113.86^\circ$ or $\arccos\left(\frac{-6}{\sqrt{22} \times \sqrt{10}}\right)$

(d) [3] (WebHW12.5 #4) Find an equation of a plane passing through $(2, 1, 0)$ and normal/orthogonal/perpendicular to $\overrightarrow{PQ}$

$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

$-2(x-2) + 3(y-1) - 3(z-0) = 0$

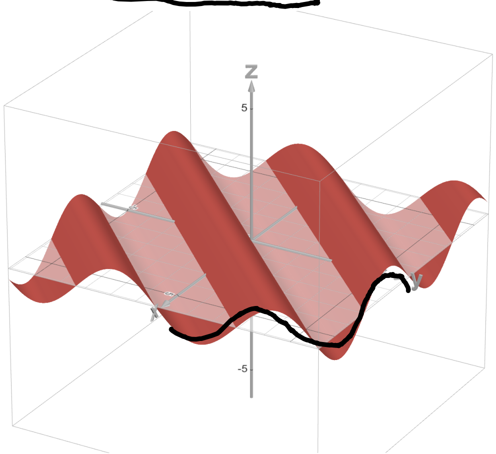
$-2x + 4 + 3y + (-3) - 3z = 0$

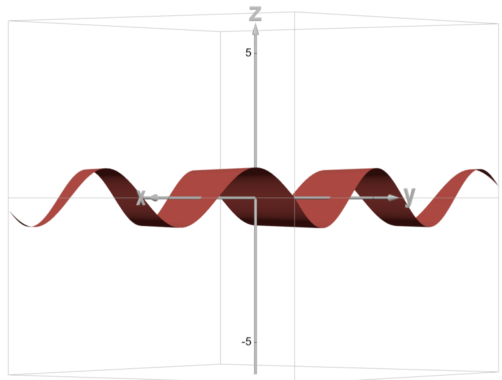
$\boxed{-2x + 3y - 3z = -1}$

3. [3] (§14.1#64) Two perspectives of the graph of $f(x, y)$ are shown below. Identify which algebraic rule below corresponds with it. Provide justification!!!

- $f(x, y) = \sin(x) - \sin(y)$
- $f(x, y) = \sin(xy)$
- $\boxed{f(x, y) = \sin(x - y)}$

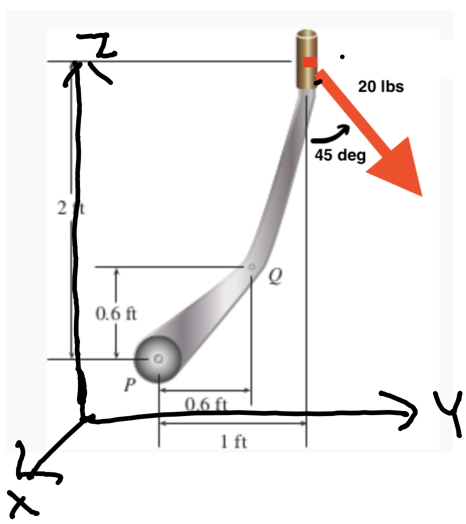
It would be sin (x-y) as then graphs show the curve fun. Also was on Web HW.





diagonal force

4. Consider the bicycle pedal shown on the right. A ~~horizontal~~ force of 20 lbs is applied to the handle as shown.



(a) [2] (3DActivity #1) Identify a 3D axis on the picture indicating the positive x, y, and z axis.

(b) [3] (WrittenHW12.4#40) Write the components of the force vector with respect to your 3D axis.

$$\boxed{\langle 0, -10, -10 \rangle}$$

(c) [3] (Quiz2#2) Find the *vector* of the torque created about the pivot point P.

$\sqrt{2^2+1^2} = \sqrt{5}$ $\quad T = r \times F \times \sin(45)$

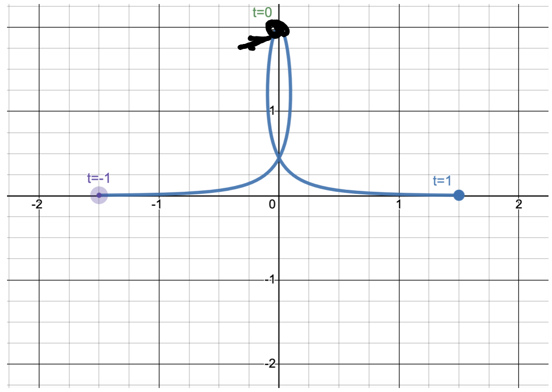
$$\boxed{\langle 0, -31.62, -31.62 \rangle}$$

$\sqrt{5} \times 10 = 23.26$ $\sqrt{5} \times 20 \times \sin(45)$

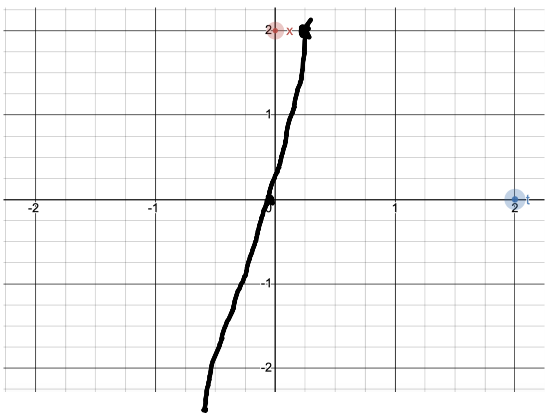
$\Rightarrow 31.62$

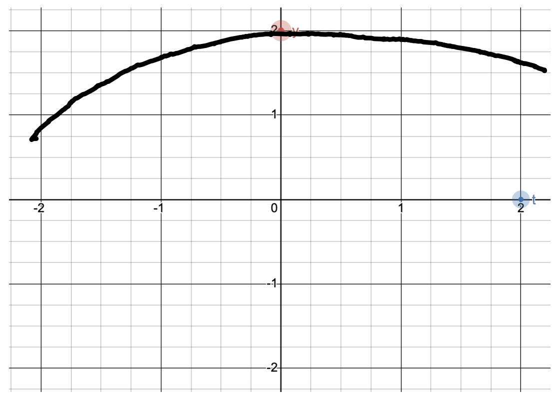
5. Consider the parametric curve $x = f(t)$, $y = g(t)$ where $-1 \leq t \leq 1$, graphed below for the following questions.

(a) [3] Looking at the graph, approximate where $\frac{dy}{dx}$ is not defined. (Report either a point on the graph or an approximate t value.)



(b) [6] (WrittenHW§10.1#32) Sketch the equations $x = f(t)$ and $y = g(t)$ on the pair of axis below.





(c) [4] (WebHW10.2#3) Given the following information, find the (approximate) line tangent to the curve $x = f(t)$, $y = g(t)$ when $t = \frac{1}{2}$. Use whatever form of a line you like (eg. parametric, slope-intercept, standard, etc)

$f\left(\frac{1}{2}\right) \approx 0$ $\quad$ $g\left(\frac{1}{2}\right) \approx .45$ $\quad$ $f'\left(\frac{1}{2}\right) \approx 1$ $\quad$ $g'\left(\frac{1}{2}\right) \approx -2.68$