Key

Quiz 3

The following trigonometric identities are provided:

$$\cos(2x) = \cos^2(x) - \sin^2(x) \qquad \sin^2(x) = \tfrac{1}{2}[1 - \cos(2x)]$$
$$\cos^2(x) = \tfrac{1}{2}[1 + \cos(2x)] \qquad \sin(2x) = 2\sin(x)\cos(x)$$

Show *all* your work. Reasonable supporting work must be shown to earn credit. There are *two* sides to this quiz.

1. [3] (WebHW7 #1) Consider the integral $\int xe^{116x}\,dx$, Identify u and dv for finding the integral using integration by parts. Do not evaluate the integral.

Start (.5)
notation (.5)

$u = x$ (+1)

if make up original +1

$dv = e^{116x}\,dx$ (+1)

notice $du = dx$ $\quad v = \frac{1}{116}e^{116x}$

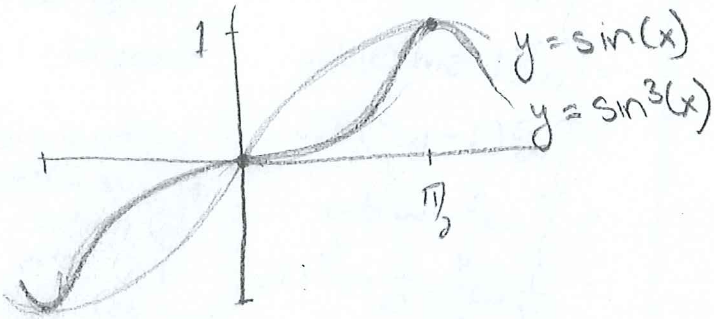
$$\int xe^{116x}\,dx = x\,\frac{1}{116}e^{116x} - \int \frac{1}{116}e^{116x}\,dx$$

to complete w/ substitution

$w = 116x$
$dw = 116\,dx$
$\frac{1}{116}\,dw = dx$

$\int e^{116x}\,dx \Rightarrow \int e^{w}\,\frac{1}{116}\,dw = \frac{1}{116}e^{w} + c$

2. [3] (§8.3 #74) Set up the definite integral(s) to compute the area trapped between $y = \sin(x)$, $y = \sin^3(x)$, $x = \frac{-\pi}{2}$, and $x = \frac{\pi}{2}$. Do not compute the answer.



$\sin^3(x)$ is on top $\qquad$ $\sin(x)$ is on top

graph +.5

(+1) break into two areas?

$$\int_{-\pi/2}^{0} \sin^3(x) - \sin(x)\,dx + \int_{0}^{\pi/2} \sin(x) - \sin^3(x)\,dx$$

top function (+1)
limits (+1)

$\frac{x^3}{x^2} = \frac{\cancel{x}\cancel{x}x}{\cancel{x}\cancel{x}} = x^1 = x^{3-2}$

$x^3 x^2 = (xxx)(xx) = x^5$

3. [4] (IP Activity #2, Trig Activity #1) Choose ONE of the following Clearly identify which of the two you are answering and what work you want to be considered for credit.

   No, doing both questions will not earn you extra credit.

   Find the indefinite integral of:

   (a) $\displaystyle\int \frac{\ln(t)}{\sqrt{t}}\, dt$

   (b) $\displaystyle\int \cos^3(\theta)\sin^3(\theta)\, d\theta$

a) $\int \frac{\ln(t)}{\sqrt{t}} dt = \int t^{-1/2} \ln(t) dt$

~~Know?~~
alg? hm...
sub? ~~$w = \ln(t)$~~
~~$dw = 1/t\, dt$~~

IP? $u = \ln(t)$ $\quad v = 2t^{1/2}$
$du = 1/t\, dt$ $\quad dv = t^{-1/2} dt$

$\int \frac{\ln(t)}{\sqrt{t}} dt = \ln(t) \cdot 2t^{1/2} - \int 2t^{1/2} \cdot \frac{1}{t} dt$

select a technique (.5)
use tech. correctly (1.5)
notation (1)
+c (.5)
got it (.5)

$= 2\sqrt{t}\ln(t) - 2\int \frac{t^{1/2}}{t} dt$

$= 2\sqrt{t}\ln(t) - 2\int t^{-1/2} dt$

$= 2\sqrt{t}\ln(t) - 2 \cdot 2t^{1/2} + c$

$= 2\sqrt{t}\ln(t) - 4\sqrt{t} + c$

Check:
$\frac{d}{dt}(2\sqrt{t}\ln(t) - 4\sqrt{t} + c)$
$= 2\sqrt{t} \cdot \frac{1}{t} + 2 \cdot \frac{1}{2} t^{-1/2} \cdot \ln t - 4 \cdot \frac{1}{2} t^{-1/2}$
$= \cancel{2t^{-1/2}} + t^{-1/2}\ln(t) - \cancel{2t^{-1/2}}$
$= \frac{\ln(t)}{\sqrt{t}}$ ✓

b) $\int \cos^3(\theta)\sin^3(\theta) d\theta$

~~Know?~~
alg? ...ug.
sub? $w = \sin\theta$
$dw = \cos\theta\, d\theta$
need to get extra $\cos^2\theta$ into $\sin^2\theta$'s

$\int \cos^3(\theta)\sin^3(\theta)d\theta = \int \sin^3(\theta)\cos^2\theta\,\underline{\cos\theta\, d\theta}$

$= \int w^3 \cos^2\theta\, dw$

since $\sin^2\theta + \cos^2\theta = 1$
$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$= \int w^3(1 - \sin^2\theta) dw$

$= \int w^3(1 - w^2) dw$

$= \int w^3 - w^5 dw$

$= \frac{1}{4}w^4 - \frac{1}{6}w^6 + c$

$= 1/4 \sin^4\theta - \frac{1}{6}\sin^6\theta + c$

(.5) select a technique
(1.5) use a technique correctly
(1) notation
(.5) plus c
(.5) got it

Check: $\frac{d}{dx}(1/4 \sin^4\theta - 1/6 \sin^6\theta + c)$
$\cancel{\frac{1}{4}} \cdot \cancel{4}\sin^3\theta \cdot \cos\theta - \cancel{\frac{1}{6}} \cdot \cancel{6}\sin^5\theta\cos\theta + 0$
$\cos\theta[\sin^3\theta - \sin^5\theta]$
$\cos\theta \sin^3\theta[1 - \sin^2\theta]$
$\cos\theta \sin^3\theta \cos^2\theta$
$\cos^3\theta\sin^3\theta$ ✓