Exam 2 TMath 125 Autumn 2019

NAME: Key

1. [7] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T F $(x^2)^3 = x^5$ $(x^2)^3 = x^2x^2x^2 = (xx)(xx)(xx) = x^6$

T F $\sqrt{b^2 + x^2} = b + x$ let $x=1$ and $b=1$ $\sqrt{1^2+1^2} = \sqrt{2} \neq 2 = 1+1$

T F $\int x^2 \cdot e^x \, dx = \frac{1}{3}x^3 \cdot e^x + c$ $\frac{d}{dx}\left(\frac{1}{3}x^3 e^x + c\right) = \frac{1}{3}x^3 e^x + e^x x^2 + 0$

T F $\frac{d}{dx}(\cos(x)) = \sin(x)$ $\frac{d}{dx}(\cos(x)) = -\sin(x)$

T F $\sec(x) = \frac{1}{\cos(x)}$ by definition

T F $\int \ln(x) \, dx = \frac{1}{x} + c$ $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

T F $\int 7^x \, dx = \frac{1}{\ln(7)} 7^x + c$ $\frac{d}{dx}\left(\frac{1}{\ln 7} \cdot 7^x + c\right) = \frac{1}{\cancel{\ln 7}} \cancel{\ln 7}\, 7^x + 0$

Show all your work. Reasonable supporting work must be shown to earn credit.

2. Let $f(x)$ be a function.

(a) [2] Explain what $\int_0^5 f(x)\,dx$ is.

The signed (+.5) area trapped between the (+.5) x-axis, (+.5) the graph of f, and between (+.5) x=0 and x=5.

By signed area, we mean the area above the x-axis is positive and the area below the x-axis is negative

(b) [2] Explain the mathematical difference between $\int f(x)\,dx$ and $\int_0^5 f(x)\,dx$.

(+1) $\int f(x)dx$ is the family of antiderivatives of $f(x)$. That is, all functions $F(x)$ so that $\frac{d}{dx}F(x) = f(x)$.

(+.5) difference

(+.5) whereas $\int_0^5 f(x)dx$ is a number corresponding to area described in (a).

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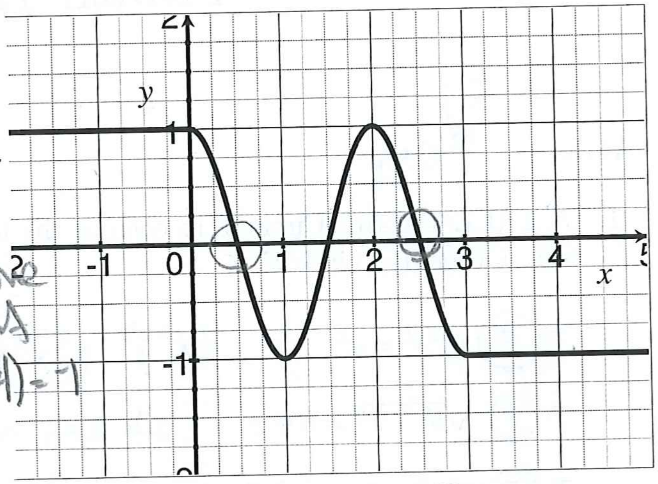
3. (§8.3 #68) The graph of $f'(x)$ is given below. Use the graph of $f'(x)$ to answer:



(a) [2] Approximate the slope of the line tangent to f at $x = 4$. Explain how you know.

+1 if find slope of $f'(x)$

(+1) -1

(+1) $f'(4)$ is by def. of derivative the slope of the line tangent to f at $x=4$, and $f'(4) = -1$

(b) [3] Find the approximate x where f reaches a maximum. Explain how you know.

+1 if find max of $f'(x)$

@ $x = \frac{1}{2}$ (+.5) and $\frac{5}{2}$ (+.5)   got one (+.5)   (+.5) slope idea

(+1) The slope increases or is positive as we approach a max of f + then switches to negative after the max. At $x = \frac{1}{2}$ + $\frac{3}{2}$ this change in slope is recorded as graph moves from + to −.

4. [5] One problem required a substitution of $x = 5\sin(\theta)$. Find the following quantities in terms of x:

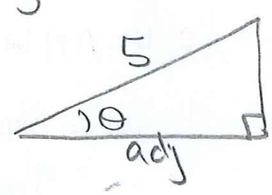
$x = 5 \sin \theta$

$\Rightarrow \frac{x}{5} = \sin \theta$

(a) $\sin(\theta) = \frac{x}{5}$ (+1)

(solve for $\sin\theta$ given)

(+1)



(b) $\cos(\theta) = \frac{\sqrt{25-x^2}}{5}$ (+1)

Sohcahtoa

$\cos\theta = \frac{adj}{hyp} = \frac{adj}{5}$

(c) $\tan\theta = \frac{opp}{adj} = \frac{x}{\sqrt{25-x^2}}$ (+1)

(+1) Note: $(adj)^2 + x^2 = 5^2$

$\Rightarrow adj = \sqrt{25-x^2}$

start +.5

use Pythagoras +.5   use correctly +.5

see Sohcahtoa +.5

def of tan +.5

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5. [10] (§8.2 #64, WebHW9, TrigActivity#1) Find the indefinite integrals for TWO of the following:

(a) $\int \cos^2(\theta)\sin^3(\theta)\,d\theta = \int u^2 \sin^2\theta \underbrace{\sin\theta\, d\theta} = -\int u^2 \sin^2\theta\, du$

$= \int u^2(1-\cos^2\theta)du = -\int u^2(1-u^2)du$

$= \int -u^2 + u^4 du = -\frac{1}{3}u^3 + \frac{1}{5}u^5 + c$

$= -\frac{1}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta + c$

Start a technique (+.5) ~~Know?~~
Use a technique (+1.5) Sub?
notation (+1)
+c (+.5)
try 2nd tech/got it (+1.5)
2nd technique (+1)

$u = \cos\theta$
$du = -\sin\theta\, d\theta$
$-du = \sin\theta\, d\theta$

Check: $\frac{d}{dx}\left(-\frac{1}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta + c\right) = +\frac{1}{3}\cdot 3\cos^2\theta\sin\theta + \frac{-1}{5}\cdot 5\cos^4\theta\sin\theta + 0$

$= \cos^2\theta\sin\theta - \cos^4\theta\sin\theta = \cos^2\theta\sin\theta(1-\cos^2\theta)$

$= \cos^2\theta\sin\theta\sin^2\theta = \cos^2\theta\sin^3\theta$ ✓

(b) $\int \sqrt{1-4x^2}\,dx = \int \sqrt{1-4(\frac{1}{2}\sin\theta)^2}\cdot\frac{1}{2}\cos\theta\, d\theta = \frac{1}{2}\int \cos\theta\cos\theta\, d\theta$

$= \frac{1}{2}\int \cos^2\theta\, d\theta = \frac{1}{2}\int \frac{1}{2}(1+\cos 2\theta)d\theta = \frac{1}{4}\int 1 - \cos 2\theta\, d\theta$

$= \frac{1}{4}\int 1\, d\theta - \frac{1}{4}\int \cos 2\theta\, d\theta$

$= \frac{1}{4}\theta - \frac{1}{4}\int \frac{1}{2}\cos w\, dw$

$= \frac{1}{4}\theta - \frac{1}{8}\sin w + c = \frac{1}{4}\theta - \frac{1}{8}\sin 2\theta + c$

$= \frac{1}{4}\theta - \frac{1}{8}\sin\theta\cos\theta + c$

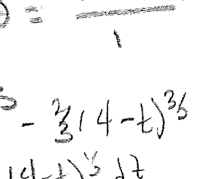
$= \frac{1}{4}\arcsin(2x) - \frac{1}{8}(2x)(\sqrt{1-4x^2}) + c$

Start a technique (+.5)
use a technique (1.5)
notation (+1)
+c (+.5)
try 2nd tech/got it (+.5)
2nd technique (+1)

~~Know?~~
~~Sub?~~
~~$u = 1-4x^2$~~
~~$du = -4\cdot 2x\,dx$~~
trig sub?
$x = \frac{1}{2}\sin\theta$
$dx = \frac{1}{2}\cos\theta\, d\theta$
$\sin^2\theta + \cos^2\theta = 1$
$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$
$\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$
$\sin 2\theta = 2\sin\theta\cos\theta$

$w = 2\theta$
$dw = 2d\theta$
$\Rightarrow \frac{1}{2}dw = d\theta$

$\frac{2x}{1} = \sin\theta$



Sohcahtoa
$?^2 + (2x)^2 = 1^2$
$? = \sqrt{1-4x^2}$
$\Rightarrow \cos\theta = \frac{\sqrt{1-4x^2}}{1}$

(c) $\int t\sqrt{4-t}\,dt = \int -(4-u)\sqrt{u}\,du$

$= -\int 4\sqrt{u} - u\sqrt{u}\,du$

$= -\int 4u^{1/2} - u^{3/2}du$

$= \int -4u^{1/2} + u^{3/2}du$

$= -4\cdot\frac{2}{3}u^{3/2} + \frac{2}{5}u^{5/2} + c$

$= \frac{-8}{3}(4-t)^{3/2} + \frac{2}{5}(4-t)^{5/2} + c$

Start a technique (+.5) ~~Know?~~
Use a technique (+1.5) Sub?
notation (+1)
+c (+.5)
try 2nd tech/got it (+1.5)
2nd tech use (+1)

$u = 4-t$
$\Rightarrow t = 4-u$
$du = -dt$
$\Rightarrow -du = dt$

OR Integration by Parts
$u = t$ $\quad v = -\frac{2}{3}(4-t)^{3/2}$
$du = dt$ $\quad dv = (4-t)^{1/2}dt$

$\int t\sqrt{4-t}\,dt = t\left(-\frac{2}{3}(4-t)^{3/2}\right) - \int -\frac{2}{3}(4-t)^{3/2}dt$

$= -\frac{2}{3}t(4-t)^{3/2} + \frac{2}{3}\int (4-t)^{3/2}dt$

$= -\frac{2}{3}t(4-t)^{3/2} - \frac{2}{3}\left(\frac{2}{5}\right)(4-t)^{5/2} + c$

Check: $\frac{d}{dt}\left(\frac{-8}{3}(4-t)^{3/2} + \frac{2}{5}(4-t)^{5/2} + c\right) = \frac{+8}{3}\cdot\frac{3}{2}(4-t)^{1/2} - \frac{2}{5}\cdot\frac{5}{2}(4-t)^{3/2} + 0$

$= 4(4-t)^{1/2} - (4-t)^{3/2} = (4-t)^{1/2}(4-4+t) = (\sqrt{4-t})t$ ✓

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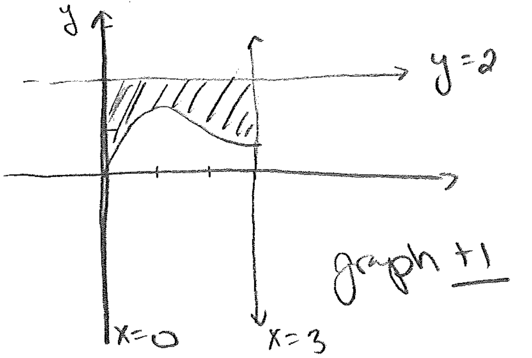
6. [3] (Quiz3 #2) Set up the definite integral(s) to compute the area trapped between $y = 2xe^{-x}$, $y = 2$, $x = 0$ and $x = 3$. Do not compute the answer.



graph +1

$$\int_0^3 2 - 2xe^{-x}\,dx$$

(+1) (+1.5) (+.5)

+.5 diff

+.5 use $2xe^{-x}$

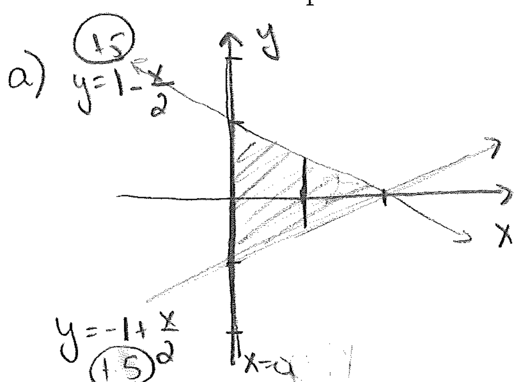
7. (Lecture) Consider a solid whose base is bounded by $y = 1 - \frac{x}{2}$, $y = -1 + \frac{x}{2}$ and $x = 0$. The cross sections perpendicular to the x-axis are equilateral triangles. Complete the following steps as you would to find the volume of the object.

(a) [2] Draw the base of the object with the x and y axis.

(b) [2] Recall the volume can be calculated by taking limits of a sum of approximating slices/sections/cylinders/shapes. Draw such an approximating slice/section/cylinder/shape that you can use to find the volume of the object. Be sure to include the x, y, and z axis.

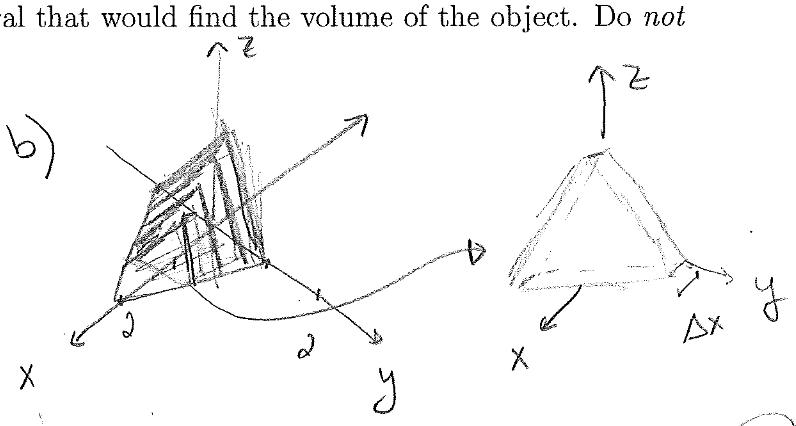
(c) [3] Set up the definite integral that would find the volume of the object. Do *not* compute this.

a)



axes (+.5) correct area (+.5)

b)



axes (+.5) nice shape that can be used (+.5)

matches (c) (+1)

c)

(+.5)

Sum area of Δ's $\cdot$ depth

(+.5)

Sum $\frac{1}{2} \cdot$ base $\cdot$ height $\cdot\ \Delta x$

Sum $\frac{1}{2}$ (dif between 2 lines) $\cdot$ height Δx

(+.5)

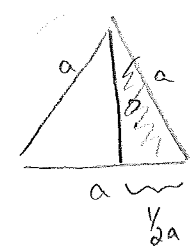
Sum $\frac{1}{2}\left(\left(1-\frac{x}{2}\right)-\left(-1+\frac{x}{2}\right)\right) \cdot$ height Δx

Sum $\frac{1}{2}(2-x) \cdot$ height Δx

(+.5) $\int_0^2 \frac{1}{2}(2-x)\frac{\sqrt{3}}{2}\cdot(2-x)\,dx$

got it (+.5)

note



$\left(\frac{1}{2}a\right)^2 + ?^2 = a^2$

$\Rightarrow ?^2 = a^2 - \frac{1}{4}a^2$

$\Rightarrow ?^2 = \frac{3}{4}a^2$

$\Rightarrow ? = \sqrt{\frac{3}{4}a^2}$

$\Rightarrow ? = \frac{\sqrt{3}}{2}a$

(+.5)

height of equilateral Δ is $\frac{\sqrt{3}}{2}\cdot$ base

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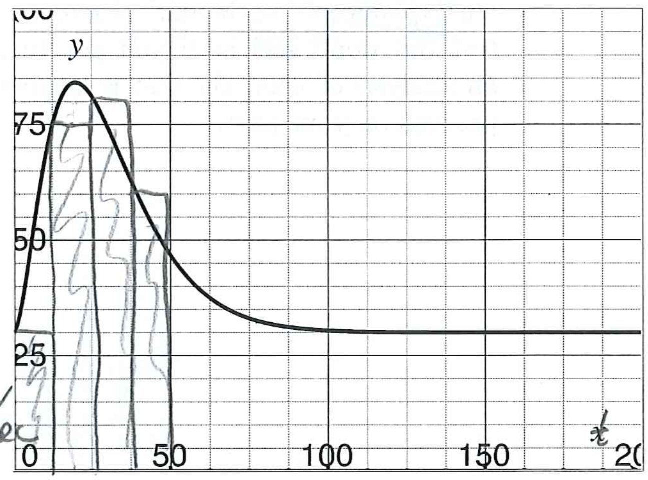
8. (Word Problem2 #4) The download rate from the internet company is variable starting low, increasing, and then decreasing again. This data download rate (megabytes/second) can be modeled by $t^2 e^{\frac{-t}{10}} + 30$ where t is seconds since the start of download. The graph is given on the right.



(a) [1] Approximate the maximum download rate.

$t \approx 20$ and rate ≈ 84 MB/sec

(the burst?)

(b) [2] Approximate how much data has been downloaded in the first 50 seconds. Specify how you are doing your approximation!

using geom shapes (+.5)

(+.5) [Using left hand rectangles (4 of them)

(+1) [$30 \cdot (12.5) + 75 \cdot 12.5 + 80 \cdot 12.5 + 60 \cdot 12.5 = 3062.5$

(c) [1] is the approximation above an over or under estimate?

(+1) [under by the looks of it ...

(d) [3] We would like to know how long it take to download a movies that is 3.5 gigabytes. Set up the equation (involving an integral) to find this time. Do not solve the equation.

(+.5)

$3.5\,\cancel{GB} \; \frac{1000\,MB}{1\,\cancel{GB}} = 3500\,MB$

(+1) (+.5)

find T such that $\int_0^T t^2 e^{-t/10} + 30\, dt = 3500$

(+1)

+.5 integral

+.5 with $t^2 e^{-t/10} + 30$

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+1 +1

9. [2] Explain one mathematical concept that your studied well while preparing for this test but don't feel as if you got to fully demonstrate. (Note, I am not asking for an analysis of what the test is lacking but rather a stunning display of mathematical prowess on your part.)

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