Exam 1 # TMath 125 Autumn 2019

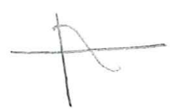
NAME: Key

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T F $\frac{1}{\frac{a}{2}} = \frac{2}{a}$ $\qquad \frac{1}{\frac{a}{2}} = 1 \div \frac{a}{2} = 1 \cdot \frac{2}{a} = \frac{2}{a}$

T F $(x^3)^2 = x^5$ $\qquad (x^3)^2 = (x^3)(x^3) = (x x x)(x x x) = x^6$

T F $\int x^2\, dx = 2x + c$ $\qquad d/dx(x^2) = 2x$ and $\int x^2 dx = \frac{1}{3}x^3 + C$

T F $\frac{d}{dx}(\cos(x)) = \sin(x)$ $\qquad$  $\quad d/dx(\cos(x)) = -\sin(x)$

T F $\sum_{i=4}^{4} i = 4$ $\qquad = 4$

T F $x^{-2} = x^{\frac{1}{2}}$ $\qquad x^{-2} = \frac{1}{x^2} \neq x^{1/2} = \sqrt{x}$

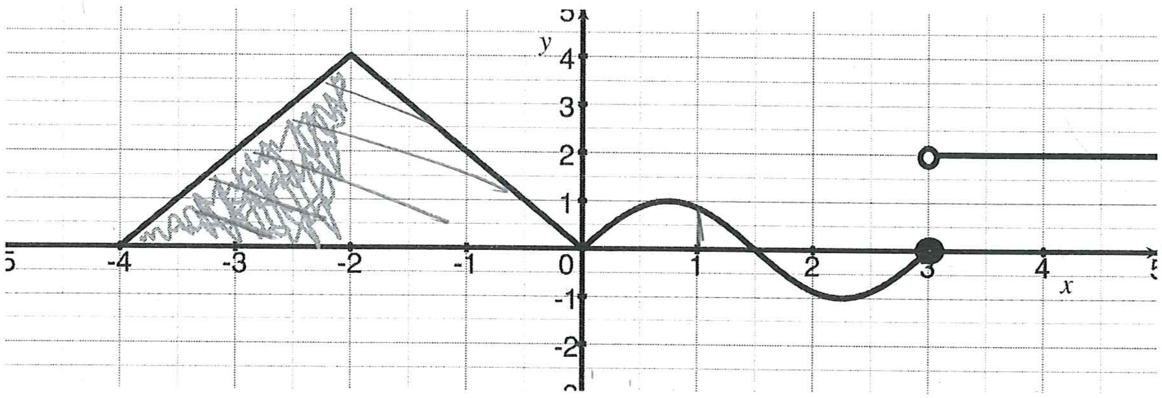
Show all your work. Reasonable supporting work must be shown to earn credit.

2. [2] What is your favorite math theorem? Why?

start (+.5) $\qquad$ (+.5)

true/correct (+.5) $\qquad$ sense (+.5)

3. For this page you will use the function f graphed below and the function g. It is given that $\int_{-4}^{-2} g(x)\,dx = 3$ and $\int_{-2}^{0} g(x)\,dx = 2$



(a) [1] (DefinateIntegral #1) Find $\int_{1}^{1} f(x)\,dx$

0

(b) [3] (Quiz1 #1) Describe the shaded area as a definite integral.

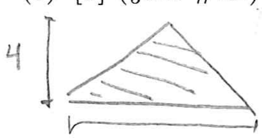
(+.5) $\int_{-4}^{-2}$ (+.5) $\underbrace{f(x)}_{(+.5)}\,dx$ or $\int_{-4}^{-2} 2x+8\,dx$

bounds (+1.5)

Slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{1}$
thru $(-2,4)$ so
$4 = 2(-2)+b$
$\Rightarrow 4 = -4+b$
$8 = b$

(c) [3] (§5.3 #52) Find $\int_{-4}^{0} f(x)\,dx$

(+1) area
(+.5) $\frac{1}{2}\cdot$ base $\cdot$ height
(+1) graph reading
(+.5) got it



$\frac{1}{2}\cdot 4\cdot 4 = 8$

OR $\int_{-4}^{-2} f(x)dx = \int_{-4}^{-2} f(x)dx + \int_{-2}^{0} f(x)\,dx$ (+.5)

$= \int_{-4}^{-2} 2x+8\,dx + \int_{-2}^{0} -2x\,dx$ (+.5)

$= \left[x^2+8x\right]_{-4}^{-2} + \left[-x^2\right]_{-2}^{0}$ (+1) (+.5)

$= (4-16)-(16-32) + (-0-4)$ got it (+.5)

$= -12+16+4 = 8$

ug... other way is better...

(d) [3] (WebHW2 #7) Find $\int_{-4}^{0} f(x) + g(x)\,dx$

$\int_{-4}^{0} f(x)+g(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_{-4}^{0} g(x)\,dx$ (+.5)

$= \underbrace{8}_{\text{from \#3c}} + \int_{-4}^{-2} g(x)\,dx + \int_{-2}^{0} g(x)\,dx$ (+.5)

notation (+.5)

$= 8 + 3 + 2$

(+.5) (+.5)

$= 13$

got it (+.5)

4. [3] (WebHW3 #1) Expand $\sum_{j=2}^{6}\left(\frac{j}{j^2-3}\right)$. (You do *not* need to compute this!)

$$\frac{2}{2^2-3} + \frac{3}{3^2-3} + \frac{4}{4^2-3} + \frac{5}{5^2-3} + \frac{6}{6^2-3}$$

do 5 terms (+.5)
numerator (+.5)
denominator (+.5)
correct terms (+.5)
start at j=2 (+.5)
~~start~~ (+.5)
notation (+.5)

5. Find the indefinite integral:

(a) [3] (antiderivatives #2) $\int \frac{x^3-4x^2}{2x}\,dx$

$$\int \frac{x^3}{2x} - \frac{4x^2}{2x}\,dx$$

fractions (+.5)
simplify (+1)

$$= \int \frac{1}{2}x^2 - 2x\,dx$$

$$= \frac{1}{2}\cdot\frac{1}{3}x^3 - x^2 + C$$ (+.5) (+.5) (+.5)

$$= \frac{1}{6}x^3 - x^2 + C$$

ck $\frac{d}{dx}\left(\frac{1}{6}x^3 - x^2 + C\right) = \frac{1}{6}3x^2 - 2x + 0$ ✓

OR

$u = 2x \Rightarrow \frac{1}{2}u = x$

$du = 2dx \Rightarrow \frac{1}{2}du = dx$

$$\int \frac{x^3 - (2x)^2}{2x}\,dx = \int \frac{x^3 - u^2}{u}\,\frac{1}{2}du$$

$$\frac{1}{2}\int \frac{\left(\frac{1}{2}u\right)^3 - u^2}{u}\,du = \frac{1}{2}\int \left(\frac{1}{2}\right)^3\frac{u^3}{u} - \frac{u^2}{u}\,du$$

$$\frac{1}{2}\int \frac{1}{8}u^2 - u\,du = \frac{1}{2}\left[\frac{1}{8}u^3\frac{1}{3} - u^2\frac{1}{2}\right]$$

$$= \frac{1}{6}\cdot\frac{1}{8}u^3 - \frac{1}{4}u^2 + C = \frac{1}{6}\cdot\frac{1}{8}(2x)^3 - \frac{1}{4}(2x)^2 + C$$

$$= \frac{1}{6}x^3 - x^2 + C$$

(b) [3] (WebHW5 #7) $\int e^{\ln(6x-5)}\,dx$

$$\int e^{\ln(6x-5)}\,dx = \int 6x - 5\,dx$$ b/c $\ln(x)$ and e^x are inverses? :)

OR

$$= 6\cdot\frac{1}{2}x^2 - 5x + C$$ (+.5) (+.5) (+.5)

simplify (+1)
notation (+.8)

$$= 3x^2 - 5x + C$$

ck $\frac{d}{dx}(3x^2 - 5x + C) = 6x - 5 + 0$ ✓

(c) [4] (§5.5 #54) $\int (3-x)7^{(3-x)^2}\,dx$

$u = (3-x)^2$

(+.5) $du = 2(3-x)(-1)dx$

$\Rightarrow -\frac{1}{2}du = (3-x)dx$

try substitution (+.5)
answer in terms of x (+.5)

$$= \int 7^{(3-x)^2}(3-x)\,dx$$

(+.5)

$$= \int 7^u\left(-\frac{1}{2}\right)du$$ (+1)

(+.5) (+.5)

$$= -\frac{1}{2}\frac{1}{\ln 7}\cdot 7^u + C$$

3

$$= \frac{-1}{2\ln 7}\cdot 7^{(3-x)^2} + C$$

OR

ck $\frac{d}{dx}\left(\frac{-1}{2\ln 7}7^{(3-x)^2} + C\right) = \frac{-1}{2\ln 7}\cdot \ln 7\, 7^{(3-x)^2}\cdot 2(3-x)(-1)$ ✓

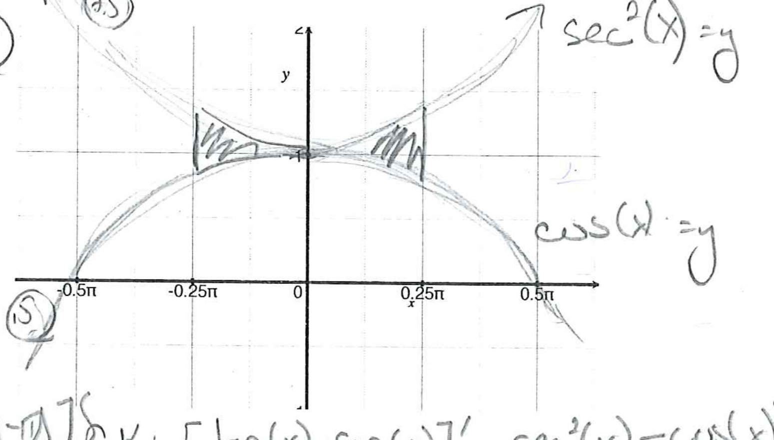
Typo

6. (§7.1 #10) Consider $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x - \cos(x)\, dx$ (+.5)

start (+.5) +.5 bounds (+1)

(a) [3] Shade the area corresponding with the definite integral.

(b) [3] Compute the area.



$\int_{-\pi/4}^{\pi/4} \sec^2(x)\cos(x)\,dx$

notation (+.5) $= \tan(x) - \sin(x)\Big]_{-\pi/4}^{\pi/4}$ (+.5) (+.5)

plug in endpts (+1) $= \left[\tan(\tfrac{\pi}{4}) - \sin(\tfrac{\pi}{4})\right] - \left[\tan(-\tfrac{\pi}{4}) - \sin(-\tfrac{\pi}{4})\right]$ CK: $[\tan(x) - \sin(x)]' = \sec^2(x) - \cos(x)$ ✓

evaluate (+.5) $= \left(1 - \frac{\sqrt{2}}{2}\right) - \left(-1 - -\frac{\sqrt{2}}{2}\right) = 1 - \frac{\sqrt{2}}{2} + 1 - \frac{\sqrt{2}}{2}$

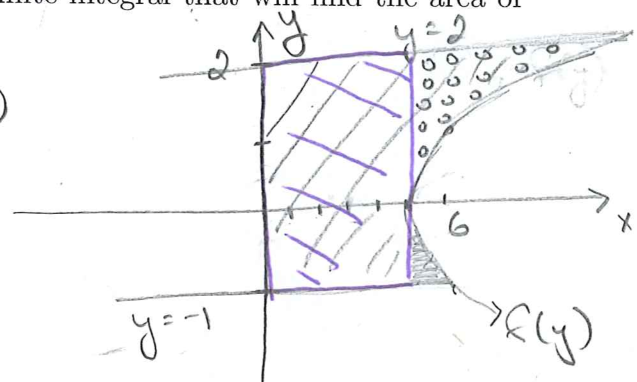
$= 2 - \sqrt{2}$ $\approx .586$

7. (WebHW6 #4) Consider the area trapped by $f(y) = y^2 + 5$, $g(y) = 0$, $y = -1$, and $y = 2$.

(a) [3] Sketch and shade the region bounded by the graphs.

(b) [4] Set up (but do *not* compute!) the definite integral that will find the area of the shaded region above.

(a) axes labeled x & y (+.5)
values for tick marks (+.5)
area idea (+.5)
horiz lines (+1)
parabola (+.5)



(b)

$\int_{-1}^{2} y^2 + 5 - 0\,dy$

OR

with respect to x (+.5) (+1)

$\sqrt{x-5} = y$

$-\sqrt{x-5} = y$

$\cancel{\int_0^5 2 - -1\,dx} + \int_5^6 -\sqrt{x-5} - -1\,dx$

$+ \int_5^9 2 - \sqrt{x-5}\,dx$ (+1)

solve for y (+1)

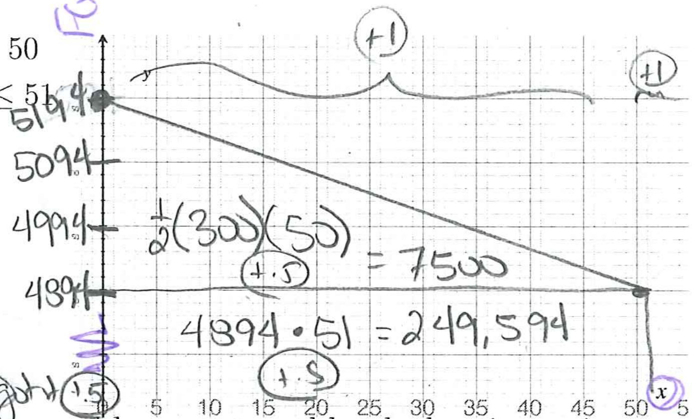
broke up area (+.5)

limits (+.5)
top function recognized (+1)
dy (+.5)
function uses y (+.5) or x consistently
~~notation (+.5)~~
start (+.5)

8. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) [5] (Word Problem1 #1or 5) A crane operator has to pull up a large bundle from the ground to the construction site that is 50 meters high. The total force (F) experienced by the crane as the bundle was brought up to the construction site as a function of x meters traveled by the bundle is given by the piece-wise defined function below:

$$F(x) = \begin{cases} -6x + 5194 & \text{if } 0 \le x < 50 \\ 4894 & \text{if } 50 \le x \le 51 \end{cases}$$

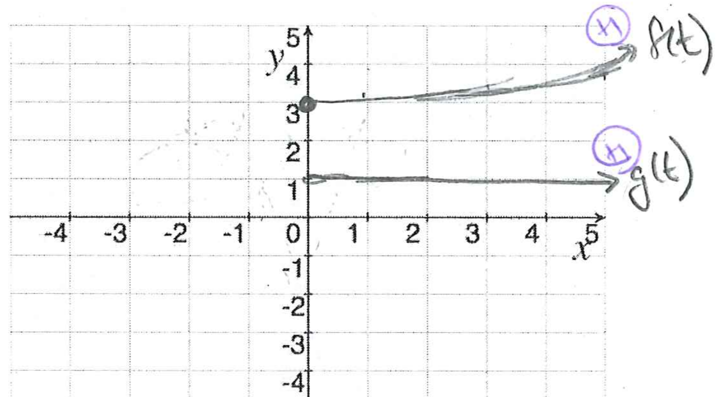


i. [3] Add appropriate (+1) labels to the vertical axis and graph $F(x)$ on the graph.

ii. [2] Compute the total work done by the crane to lift the bundle. 257,094 J got it (+.5)

Work = Area below + force graph w.r.t. distance (+.5)

(b) [5] A brokerage account is being managed by parents and has had an increasing rate of growth over that time. Generally the account has increased by $f(t) = 3 * e^{.03t}$ dollars t years after the account was opened. Let $g(t) = 1$ record the yearly fee charged by the parents.



i. [2] Graph $f(t)$ and $g(t)$. do

ii. [2] Approximate the amount of money earned in the first three years. Clearly indicate your methods. → not net

iii. [1] Explain what $\int_0^3 f(t) - g(t)\,dt$ means in real world terms. (+.5) area

ii) $\int_0^3 f(t)\,dt = \int_0^3 3 \cdot e^{.03t}\,dt$ (+1)

$u = .03t$
$du = .03\,dt$
$\frac{100}{3}\,du = dt$

$= \int_0^{.09} 3 e^u \frac{100}{3}\,du$

$= 100 \int_0^{.09} e^u\,du = 100 e^u \Big]_0^{.09} = 100e^{.09} - 100 \approx \$9.42$ got it (+.5)

iii) The net money added to the account ($ earned − fees)

9. [1] What concept did you study but not see on the exam?

mean value theorem

Key