TMath 124

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit. Recall that you are allowed to use a one-sided 8.5in by 11in sheet of physical (not digital!) notes for this exam.
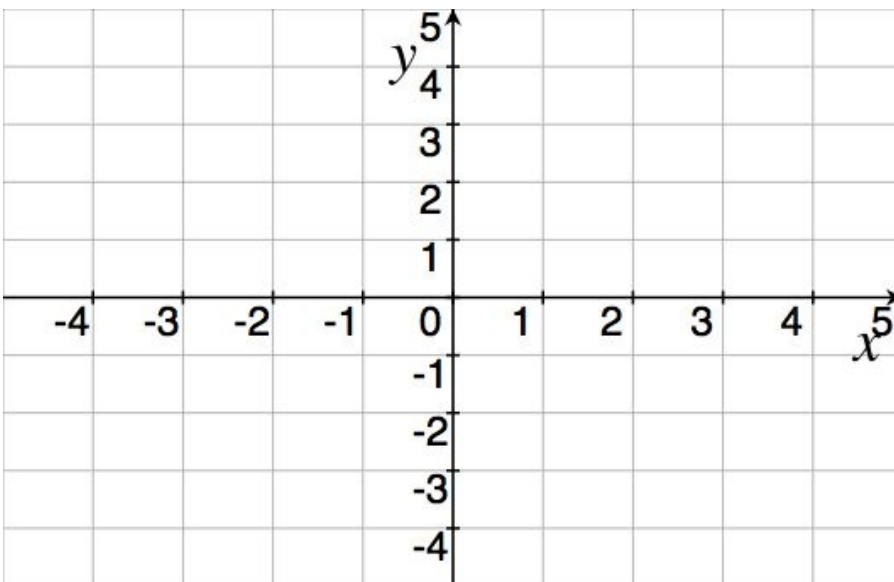
1. [5] Sketch the graph of an example function f that satisfies the following conditions:

   (a) $\lim\limits_{x\to\infty} f = 2$

   (b) $\lim\limits_{x\to-3} f(x) \neq f(-3)$

   (c) $f'(0) = -2$

   (d) f is continuous on $(-3, \infty)$.

2. [3] Sketch the line tangent to the function f you drew in number 1 when $x = 0$. Find the equation of the tangent line.

3. [4] The work for the following problems is *wrong*. Explain why the solution is wrong and then find the correct solution. "Find y' given $y = 5\cos(\pi x)\cos(x)$."

$$y' = [5\cos(\pi x)\cos(x)]'$$
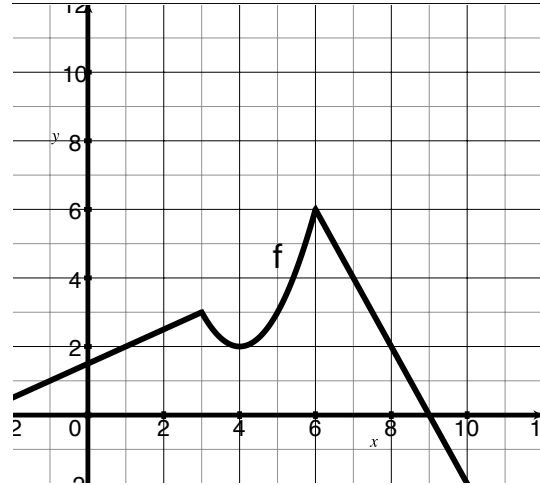$$y' = 5[\cos(\pi x)\cos(x)]'$$
$$y' = 5(-\sin(\pi x) \cdot [\pi x]') \cdot (-\sin(x))$$
$$y' = 5 + \sin(\pi x) \cdot \pi \sin(x)$$
$$y' = 5 + \pi \sin(\pi x) \cdot \sin(x)$$

4. [12] Let the graph of f and g be those shown below.

$$g(x) = \begin{cases} -\frac{1}{2}x & \text{if } x < 0 \\ 4 & \text{if } x = 0 \\ \log_4(x+1) & \text{if } 0 < x \end{cases}$$

Estimate the following (if they exist):

$f(1)$

$(f - 2g)'(0)$

$(f \cdot g)'(-\frac{1}{2})$

$\frac{d}{dx}(f(g(x)))|_{x=4}$

All x values where $f'(x) = 0$

5. [8] Find the derivatives of the following and do *not* simplify.

$$y = \frac{\arccos(x)}{5x^2 + x - 6}$$

$$y = (x^5 + 4x - 2)^{2x+1}$$

6. [6] The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3m from the wall, it slides away from the wall at a rate of 0.2m/s. How long is the ladder?

7. Porche 911 can go from 0 to 60 mph in 5.6 seconds. The distance that the car can travel in the first few seconds can be given by the function $d(t) = 7.85t^2$

   (a) [1] Find the average velocity of the car between $t = 2.5$ and $t = 3$.

   (b) [3] Find the instantaneous velocity of the car after $t = 3$ seconds.

   (c) [3] Does the Porche 911 ever reach 60 miles per hour per hour in the first 5.6 seconds? Justify your answer.

   (d) [1] Find a function to describe the acceleration at t seconds.

8. [4] Choose a problem from this exam that you've already answered,

   (a) show a second way of approaching/building a solution, and

   (b) explain why you did not choose this second method initially.