TMath 124

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit. Recall that you are allowed to use a one-sided 8.5in by 11in sheet of physical (not digital!) notes for this exam.

1. [8] Determine which derivative rule(s) you will use to find $\frac{dy}{dx}$ and find $\frac{dy}{dx}$

	Derivative Rule(s)		$\frac{dy}{dx}$
$x^2 + \arctan(x^2)$			
$y = \dfrac{x^2 \log_3(x)}{x}$			

2. The work for the following problems is *wrong*. Explain why the solution is wrong and then find the correct solution.

   (a) [4] Find $\frac{dy}{dx}$ given $y = (x-5)^{\frac{1}{x}}$.

   $$\ln(y) = (x-5)^{\frac{1}{x}} \ln$$
   $$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\ln(x-5)$$
   $$\frac{dy}{dx} = y \cdot \frac{1}{x}\ln(x-5)$$

   (b) [4] Find y' given $y = 5\cos^4(\pi x)$.

   $$y' = [5\cos^4(2x)]'$$
   $$y' = 5 - \sin^4(2x) \cdot [(2x)^4]'$$
   $$y' = 5 - \sin^4(2x) \cdot 4(2x)^3[2x]'$$
   $$y' = 5 - \sin^4(2x) \cdot 4(2x)^3 \cdot 2$$

3. Let f be the function graphed on the right. Let $h(x) = 2^x$. Using this and the information about g given below, find the following (if possible!):
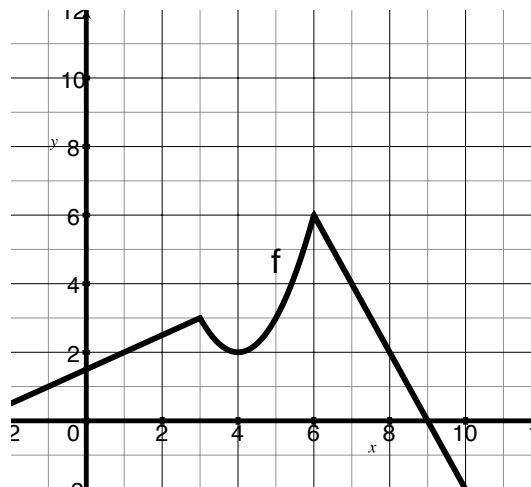
$g(2) = 1 \qquad g'(2) = -6$

$g(4) = 3 \qquad g'(4) = -5$

(a) [1] Find $f(8)$

(b) [1] Find $f'(6)$

(c) [3] Find $\frac{d}{dx}\left(f(g(x))\right)|_{x=2}$.

(d) [3] Find $\left(\frac{f}{h}\right)'(4)$.

(e) [3] Find $(g \cdot h)'(4)$.

(f) [2] Find the equation of the line tangent to g at $x = 2$.

(g) [2] Use linear approximation to estimate $g(2.1)$.

4. Consider the graphs recording the displacement of Dr. Ryan Card and Dr. Julie Eaton during a short race. The following graphs their respective *displacement*, measured in feet at time t in seconds.

Ryan's displacement

Julie's displacement
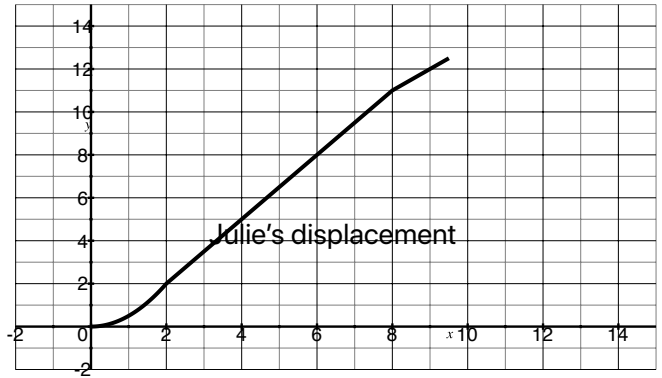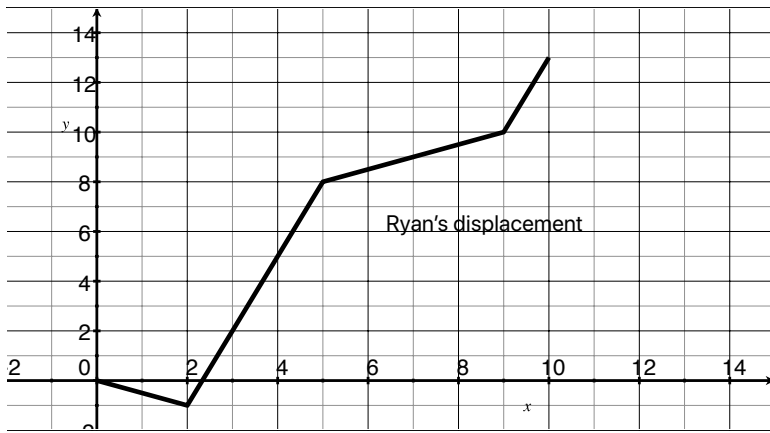
(a) [3] Who can run faster? What is his/her top speed?

(b) [2] Explain why the displacement is not increasing at a constant rate for Dr. Ryan Card.

(c) [2] If the race is 12 feet long, who wins and at what time?

5. [3] Consider $x^2 + y^2 = 100$. Find the x coordinant(s) on this graph whose tangent lines are parallel to $4y = 3x + 8$.

6. An air traffic controller spots two planes at the same altitude converging on a point, P as they fly at right angles to each other. Airplane A, is north of point P and flying south at 280 miles per hour. The second airplane, B, is east of P and is flying west at 960 miles per hour.

    (a) [1] Draw a picture of the situation described.

    (b) [2] Find a *relationship* between the rate of change of the distance between the planes and the variables you create in (a).

    (c) [2] Find the rate the distance between the planes is decreasing when airplane A is 70 miles north of P and airplane B is 240 miles east of P.

7. [4] Choose a problem from this exam that you've already answered,

    (a) show a second way of approaching/building a solution, and

    (b) explain why you did not choose this second method initially.